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Lattice Boltzmann method for slip flow heat transfer in circular microtubes: Extended Graetz problem

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ABSTRACT

Slip flow heat transfer in circular microtubes is of fundamental interest and practical importance. However, to the best knowledge of the present author, there is no open publication of developing simple and efficient lattice Boltzmann (LB) models on such topic. To bridge the gap, in this paper a simple LB model, which is based on our recent work [S. Chen, J. Tölke, M. Krafczyk, Simulation of buoyancy-driven flows in a vertical cylinder using a simple lattice Boltzmann model, Phys. Rev. E 79 (2009) 016704], is designed. In addition, the recently developed Langmuir slip model [S. Chen, Z.W. Tian, Simulation of thermal micro-flow using lattice Boltzmann method with Langmuir slip model, Int. J. Heat Fluid Flow 31 (2010) 227–235], which possesses a clear physical picture and keeps the Reynolds analogy, is extended to capture velocity slip as well as temperature jump in microtubes. The feasibility and capability of the present model are validated by the extended Graetz problem, which is a benchmark prototype for forced convection heat transfer in circular microtubes.

1. Introduction

In recent years research in the field of thermofluids at the microscale level has been constantly increasing, due to the rapid growth in technological applications which require high rates of heat transfer in relatively small spaces and volumes, such as in micro-electromechanical systems (MEMS) and very large-scale integration (VLSI) technologies [1]. In most practical applications, circular microtubes are used as popular tools to cool such electronic devices. The requirement of more efficient electronic cooling methods emphasizes the need to comprehend the heat transfer characteristics of fluids in microtubes [2]. As the size of a circular tube is reduced, the usually used no-slip boundary conditions for macroflows need to be modified so that velocity slip and temperature jump may occur on the wall. The slip boundary condition may be used when gases are at low pressure or for flow in extremely small passages. The rarefaction effects of a gas are included by the Knudsen number Kn, the ratio of the mean free path to the characteristic length in the flow field. Karniadakis and Beskok [3] have proposed the range for the Knudsen number in slip flow regime as $0.001 < \mathrm{Kn} < 0.1$. Since the different mechanisms behind the complicated phenomena in such devices are strongly intertwined, the experimental study of a single mechanism is a difficult even impossible task. Therefore, numerical simulations are attractive as they provide a controllable way to change a single property of fluid while keeping the others unchanged. Generally, the direct simulation Monte Carlo (DSMC) method can be used to simulate rarefied gas flow with heat transfer. However unfortunately, the statistical scattering and computational requirements (memory and CPU) makes DSMC inefficient, especially for low-speed, low-Kn flows [4].

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Recently, the mesoscopic lattice Boltzmann (LB) method developed from kinetic theory has been popularly applied to study microfluidic flows [5–9], to cite only a few. Compared with DSMC schemes, the LB method is more efficient intuitively in computation. More important, since the LB method is based on the continuous Boltzmann equation which may capture nonequilibrium gas flows in the whole range of Kn, it has been recognized to be a promising tool for microscale gas slip flows [10]. But discouraged, until now all existing studies with the aid of LB method are limited in flows with temperature differences inside planar microchannels. To the best knowledge of the present author there is no open literature using the LB method on slip flow heat transfer in circular microtubes.

The aim of this work is to propose an efficient and simple LB model as well as corresponding boundary scheme to simulate slip flow heat transfer in circular microtubes. The extended Graetz problem [11–15], which is a benchmark test for slip flow heat transfer in circular tubes, is adopted to validate the performance of the present model.

2. Extended Graetz problem

The Graetz problem is a simplified case of the problem of forced convection heat transfer in a circular tube in laminar flow. The steady-state hydrodynamically developed flow with constant temperature T_0 enters into the circular tube as Fig. 1 shown. The fluid temperature would change from the value T_0 at the entrance to the value T_w on the walls. With the assumptions of steady and incompressible flow, constant fluid properties, no swirl component of velocity, fully developed velocity profile, and negligible energy dissipation effects, Graetz [16] originally solved this problem analytically. However, the original solution by Graetz is valid only for continuum flow $\mathrm{Kn} < 10^{-3}$. Recently, Barron et al. [11] extended the Graetz problem to slip regime as a prototype for gas slip heat transfer in circular microtubes (so named as “extended Graetz problem”). Later, the extended Graetz problem becomes a “hot” research topic. Following the pioneer work conducted by Barron and his cooperators, a lot of excellent investigations have been done on this prototype to reveal the special but elementary heat transfer characteristics of fluids in microtubes [12–15,17–19].

Under cylindrical coordinates, the governing equations for the extended Graetz problem read [11,15,19]:

$$\frac{1}{r}\left[\frac{d}{dr}\left(r\frac{du}{dr}\right)\right] = \frac{1}{\mu}\frac{dP}{dx}, \tag{1}$$

$$u\frac{\partial T}{\partial x} = \alpha_T\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial x^2}\right], \tag{2}$$

where u and v are the axial and radial velocities; P is the pressure; T is the temperature and α_T is the thermal conductivity. $\mu = \rho v$ is the dynamic viscosity and ρ, v are the density and kinematic viscosity respectively.

In order to capture the velocity slip and temperature jump, usually the Maxwell slip model are employed to treat the boundary conditions [11–14,19]. Recently, Myong et al. [15] developed a new slip model, which inspired by the theory of adsorption phenomena pioneered by Langmuir (so named as “Langmuir slip model”) [20], to describe slip phenomena at boundaries. Compared with the Maxwell slip model, the Langmuir slip model possesses a clearer physical picture and keeps the Reynolds analogy [15,20].

3. Lattice Boltzmann model

The LB model developed in our recent work [21] can be used directly to solve the governing equations for the extended Graetz problem with its vorticity-streamfunction form:

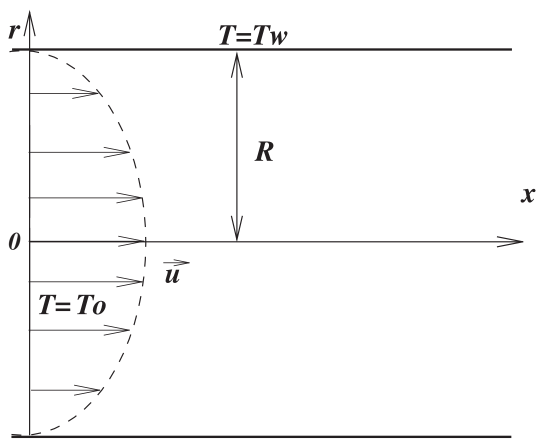

Fig. 1. Definition sketch.