Applied Mathematics and Computation 217 (2010) 3429–3441

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# Free vibration analysis of stepped beams by using Adomian decomposition method

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A R T I C L E I N F O

*Keywords:*
Adomian decomposition method
Vibration analysis
Stepped beam
Natural frequency
Mode shape

A B S T R A C T

The Adomian decomposition method (ADM) is employed in this paper to investigate the free vibrations of a stepped Euler–Bernoulli beam consisting of two uniform sections. Each section is considered a substructure which can be modeled using ADM. By using boundary condition and continuity condition equations, the dimensionless natural frequencies and corresponding mode shapes can be easily obtained simultaneously. The computed results for different boundary conditions, step ratios and step locations are presented. Comparing the results using ADM to those given in the literature, excellent agreement is achieved.



## 1. Introduction

Stepped beam-like structures are widely used in various engineering fields, such as for robot arms, in tall buildings, etc. The free vibration analysis of stepped beams has been investigated by many researchers [1–8] with great success. Numerical methods such as finite element [1,2], finite difference [3] and differential quadrature [4] or analytical methods based on fourth order differential equations [5–8] have been used in solving free vibration problems of such structures. Refs. [5,7] give an exhaustive literature survey on the free vibration analysis of stepped beams.

In this paper, a relatively new computed approach called the Adomian decomposition method [9–14] is used to analyze the free vibration problem for a stepped beam consisting of two uniform sections with arbitrary boundary conditions. The Adomian decomposition method (ADM) is a useful and powerful method for solving linear and nonlinear differential equations. The goal of ADM is to find the solution of linear and nonlinear, ordinary or partial differential equation without dependence on any small parameter as is the case with the perturbation method. In ADM the solution is considered as a sum of an infinite series, and rapidly converges to an accurate solution [10]. Recently, ADM has been applied to the problem of vibration of structural and mechanical systems [11–15].

Using the ADM, the governing differential equation for each section of the stepped beam becomes a recursive algebraic equation. The boundary conditions and continuity conditions become simple algebraic frequency equations which are suitable for symbolic computation. Thereafter, after some simple algebraic operations on these frequency equations, we can obtain the natural frequency and corresponding closed-form series solution of the mode shape simultaneously. Finally, some numerical examples are given to demonstrate the feasibility of the proposed method.

## 2. ADM for a stepped beam

Consider the free vibration of a straight Euler–Bernoulli beam consisting of two uniform sections elastically restrained at both ends, as shown in Fig. 1. The stepped beam is divided into two sections with the two mirror systems of reference $x_1$ and

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doi:10.1016/j.amc.2010.09.010

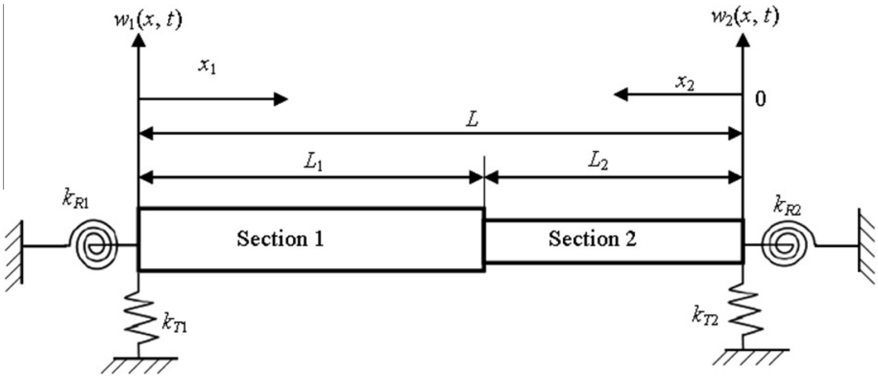


**Fig. 1.** The coordinate system for a stepped beam, elastically restrained at both ends.

$x_2$. The positive direction of the spatial coordinate $x_1$ is defined in the direction to the right for Section 1, and $x_2$ is defined in the direction to the left for Section 2.

The partial differential equation describing the free vibration in each section is as follows

$$\frac{\partial^4 w_j(x_j,t)}{\partial x_j^4}+\frac{m_j}{E_j I_j}\frac{\partial^2 w_j(x_j,t)}{\partial t^2}=0 \quad x_j\in[0\ L_j]\ (j=1,2), \tag{1}$$

where subscript $j$ = 1 and 2 denote Sections 1 and 2 of the stepped beam, respectively. $E_j$ is Young's modulus, $I_j$ is the cross-sectional moment of inertia of the beam $I_j=\frac{b_j h_j^3}{12}$, $m_j=\rho_j b_j h_j$ is the mass per unit length. And $L_j$, $b_j$, $h_j$ and $\rho_j$ is the length, width, thickness and density of each section.

According to modal analysis approach (for harmonic free vibration), the $w_j(x_j,t)$ can be separated in space and time:

$$w_j(x_j,t)=\Phi_j(x_j)e^{i\omega t}, \tag{2}$$

where $\Phi_j(x_j)$ and $\omega$ are the structural mode shape and the natural frequency, respectively. $i=\sqrt{-1}$.

Substituting Eq. (2) into (1) and separating variables for time $t$ and space $x_j$, the ordinary differential equation for each section of the stepped beam can be obtained

$$\frac{d^4\Phi_j(x_j)}{dx_j^4}-\frac{m_j\omega^2}{E_j I_j}\Phi_j(x_j)=0. \tag{3}$$

Eq. (3) can be rewritten in dimensionless form,

$$\frac{d^4\Phi_j(X_j)}{dX_j^4}-\Omega_j^4\Phi_j(X_j)=0, \tag{4}$$

where

$$X_j=\frac{x_j}{L},\Phi_j(X_j)=\frac{\Phi_j(x_j)}{L},\Omega_j^4=\frac{m_j\omega^2 L^4}{E_j I_j}.$$

Clearly,

$$\Omega_2^4=\frac{m_2}{m_1}\frac{E_1 I_1}{E_2 I_2}\Omega_1=\mu\Omega_1^4, \tag{5}$$

where $\mu=\frac{m_2}{m_1}\frac{E_1 I_1}{E_2 I_2}$, $\Omega_1$ is the dimensionless natural frequency, and the $n$th natural frequency is denoted as $\Omega_1(n)$.

According to ADM [9–14], $\Phi_j(X_j)$ in Eq. (5) can be expressed in terms of an infinite series

$$\Phi_j(X_j)=\sum_{m=0}^{\infty}\Phi_j^{[m]}(X_j), \tag{6}$$

where the component function $\Phi_j^{[m]}(X_j)$ will be determined recurrently.

Imposing a linear operator $G=\frac{d^4}{dX^4}$, the inverse operator of $G$ is then a 4-fold integral operator defined by

$$G^{-1}=\int\int\int\int(\ldots)dXdXdXdX \tag{7}$$