Applied Mathematics and Computation 217 (2010) 3591–3595

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

An efficient algorithm for solving generalized pantograph equations with linear functional argument

Elçin Yusufoğlu

Department of Mathematics, Dumlupinar University, Art-Science Faculty, 43100 Kutahya, Turkey

ARTICLE INFO

Keywords:
Advanced pantograph equation
Homotopy method
Embedding parameter

ABSTRACT

A numerical method for solving the generalized (retarded or advanced) pantograph equation under initial value conditions is presented. To display the validity and applicability of the numerical method four illustrative examples are presented. The results reveal that this method is very effective and highly promising when compared with other numerical methods, such as Adomian decomposition method, spline methods and Taylor method.

1. Introduction

Functional–differential equations with proportional delays are usually referred as pantograph equations or generalized equations. The name *pantograph* originated from the study [1] by Ockendon and Tayler. These equations arise in industrial applications and in studies based on biology, economy, control theory and electrodynamics, among others.

Properties of the analytic solution of these equations as well as numerical methods have been studied by several authors [2–4].

In recent years, there has been a growing interest in the numerical treatment of pantograph equations of the retarded and advanced type. A special feature of this type is the existence of compactly supported solutions [5]. Pantograph equations are characterized by the presence of a linear functional argument and play an important role in explaining many different phenomena. In particular they turn out to be fundamental when ODEs-based model fail. This phenomena is studied in [6–8], and has direct applications to approximation theory and wavelets [9–11].

One of the newest analytical methods to solve the mathematical problems is using both homotopy and perturbation methods. By means of generalizing the traditional homotopy method; more details about homotopy technique and its applications are found in literature [12,13]. In recent years, the applications of the homotopy perturbation method (HPM) in nonlinear problems have been devoted by scientists and engineers [14–26]. This paper, applies the HPM to the discussed problem.

2. Basic idea of HPM

To convey an idea of the HPM, we consider a general equation of the type:

$$A(u) - f(r) = 0, \tag{2.1}$$

with boundary conditions

E-mail address: eyusufoglu@dumlupinar.edu.tr

0096-3003/$ - see front matter
doi:10.1016/j.amc.2010.09.005

$$B\left(u,\frac{\partial u}{\partial n}\right)=0,\quad r\in\Gamma, \tag{2.2}$$

where $A(u)$ is defined as follows:

$$A(u)=L(u)+N(u). \tag{2.3}$$

Homotopy perturbation structure is shown as the following equation:

$$H(v,\varepsilon)=L(v)-L(u_0)+\varepsilon L(u_0)+\varepsilon[N(v)-f(r)]=0, \tag{2.4}$$

where

$$v(r,\varepsilon):\Omega\times[0,1]\to R.$$

Obviously, using Eq. (2.4) we have

$$H(v,0)=L(v)-L(u_0)=0,\quad \text{and}\quad H(v,1)=A(v)-f(r), \tag{2.5}$$

where $\varepsilon\in[0,1]$ is embedding parameter and u_0 is the first approximation that satisfies the boundary condition. The process of changes in ε from zero to unity is that of $v(r,\varepsilon)$ changing from u_0 to $u(r)$. We consider v, as following:

$$v=\sum_{i=0}^{\infty}\varepsilon^i v_i=v_0+\varepsilon v_1+\varepsilon^2 v_2+\cdots, \tag{2.6}$$

and the best approximation for solution is

$$u=\lim_{\varepsilon\to1}v=\lim_{\varepsilon\to1}\sum_{i=0}^{\infty}\varepsilon^i v_i=v_0+v_1+v_2+\cdots \tag{2.7}$$

The convergence of the method is discussed in studies [14,17].

3. Numerical examples

In this section, several numerical examples are given to illustrate the properties of the method and all of them are performed on the computer using a program written in Maple10.

Example 1. Let us first consider the equation (see [8])

$$u'(x)=\frac{1}{2}e^{\frac{x}{2}}u\left(\frac{x}{2}\right)+\frac{1}{2}u(x),\quad 0\leqslant x\leqslant 1,\quad u(0)=1, \tag{3.1}$$

which has the exact solution $u(x)=e^x$.

By HPM, we may choose a convex homotopy such as the $H(u,p)$ with components

$$H(u,p)=u'(x)-p\left(\frac{1}{2}e^{\frac{x}{2}}u\left(\frac{x}{2}\right)+\frac{1}{2}u(x)\right)=0. \tag{3.2}$$

Substituting (2.6) into (3.2), and equating the terms with identical powers of p, we have

$$p^0:\quad u_0'(x)=0,\quad u_0(0)=1, \tag{3.3}$$

$$p^k:\quad u_k'(x)=\frac{1}{2}e^{\frac{x}{2}}u_{k-1}\left(\frac{x}{2}\right)+\frac{1}{2}u_{k-1}(x),\quad u_k(0)=0. \tag{3.4}$$

Therefore, the approximate solution of Example 1 can be readily obtained by

$$u(t)=\sum_{k=0}^{\infty}u_k(x). \tag{3.5}$$

In practice, all terms of series (3.5) cannot be determined and for this reason, we use an approximation of the solution by the following truncated series:

$$\varphi_m(x)=\sum_{k=0}^{m-1}u_k(x). \tag{3.6}$$

Using the above iteration formulas (3.3) and (3.4), we can directly obtain the other components as follows

$$u_0(x)=1, \tag{3.7}$$

$$u_1(x)=\frac{1}{2}e^x+\frac{1}{2}x-1, \tag{3.8}$$

$$u_2(x)=\frac{2}{3}e^{\frac{3}{4}x}+\frac{1}{4}e^{\frac{1}{2}x}x-\frac{1}{2}e^{\frac{1}{2}x}+\frac{1}{8}x^2-\frac{1}{2}x-\frac{1}{6}, \tag{3.9}$$