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A Galerkin boundary element method based on interpolatory Hermite trigonometric wavelets

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ABSTRACT

A Galerkin boundary element method based on interpolatory Hermite trigonometric wavelets is presented for solving 2-D potential problems defined inside or outside of a circular boundary in this paper. In this approach, an equivalent variational form of the corresponding boundary integral equation for the potential problem is used; the trigonometric wavelets are employed as trial and test functions of the variational formulation. The analytical formulae of the matrix entries indicate that most of the matrix entries are naturally zero without any truncation technique and the system matrix is a block diagonal matrix. Each block consists of four circular submatrices. Hence the memory spaces and computational complexity of the system matrix are linear scale. This approach could be easily coupled into domain decomposition method based on variational formulation. Finally, the error estimates of the approximation solutions are given and some test examples are presented.

1. Introduction

It is well-known that the boundary element method (BEM) has been recognized as a powerful tool for treatment of boundary value problems in science and engineering. The main advantage of the BEM is that the dimensionality of the problem can be reduced by one. However, the computational complexity of the system matrix which is dense eliminates its advantage. Hence this drawback makes it difficult to apply the BEM to large−scale problems. To overcome the computational difficulty involved in the BEM, many fast methods have been developed in recent years. The methods like the fast multipole method [1], the panel clustering technique [2] and the wavelet BEMs [3–12] reduce the computational complexity largely.

Among the various wavelet BEMs, those based on Galerkin scheme may be attractive because of the high matrix compression rate and the easy implementation of boundary conditions. In the wavelet Galerkin BEMs, the wavelet functions are used as the trial and test functions of variational formulation. Due to the local support and the vanishing moment property of the wavelets, most of the matrix entries have small values in the wavelet Galerkin BEMs, therefore, the small entries can be truncated by a special technique to obtain a sparse matrix, but it may lead to potentially high computational cost.

Many efforts have been devoted to improving the wavelet BEMs. Quak [13] has constructed a multiresolution analysis (MRA) of nested subspaces of Hermite trigonometric wavelets. The authors [14–17] have used the trigonometric wavelets in the boundary element analysis based on the natural boundary integral equations. The purpose of this paper is to simplify the computation of the matrix entries by using the trigonometric wavelets as the basis. We call this approach as trigonometric wavelet Galerkin BEM (TWGBEM). In this approach, the kernel function of the BIE is expanded to a Fourier series. The $(2^{J+2}+1)\times(2^{J+2}+1)$ system matrix can be decomposed into a $(J+2)\times(J+2)$ block matrix, where J is the scaling level of wavelets. The analytical formulae of the matrix entries are obtained by calculating double integrals directly. It can be found

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that most of the entries are naturally zero without any truncation technique. As a result, these are only $J + 4$ non-zero sub-matrices which consist of smaller symmetric circular matrices or antisymmetric circular matrices. Hence the memory spaces and computational complexity of the system matrix are linear scale.

Besides the conventional BEM and Galerkin BEM (GBEM), there are still many other methods and techniques existing in the literature for solving the potential problems, such as the finite element method (FEM), the element free Galerkin method (EFGM) [18], the Galerkin boundary node method (GBNM) [19], the natural boundary integral equation method [20] and the wavelet Galerkin BEMs. Comparing with these methods, the present method has several advantages:

(i) Comparing with the domain type methods (such as FEM and EFGM), the present method can reduce the dimensionality of the problem by one. Thus the method is especially suitable for the problems with an unbounded domain as all boundary type methods.
(ii) Comparing with the boundary type methods, such as the GBNM, the GBEM and natural boundary integral equation method, in which the system matrices are dense, the system matrix in the present method is a block diagonal matrix and the computational complexity is reduced to linear scale from quadratic scale without any loss of accuracy.
(iii) In general, we need special truncation strategies for obtaining sparse system matrix in other wavelet Galerkin BEMs, which lead a truncation error, while the system matrix in the present method is a block diagonal matrix without any truncation strategy, thus there is no any truncation error. Therefore, the present method has higher accuracy than other wavelet Galerkin BEMs.
(iv) Because of the complicated expressions of the wavelets, the calculation of the matrix entries is difficult and time-consuming in other wavelet Galerkin BEMs. However, we can calculate analytically the matrix entries in present method and the analytical formulae are simple. Therefore, the computational cost of the system matrix is lower than other wavelet Galerkin BEMs.

The rest of this paper is outlined as follows. In Section 2, we introduce Quak's interpolatory Hermite trigonometric wavelets and their properties to be used later. Section 3 gives a brief description of the BIE and its Galerkin variational formulation for 2-D potential problems. Then, a detailed numerical implementation of the TWGBEM is described and the calculation formulae of the matrix entries are provided in the next section. In Section 5, several error estimates of the approximate solutions are deduced. Section 6 provides some numerical tests on theoretical results of the proposed method. Finally, the conclusions are given in Section 7.

2. Interpolatory Hermite trigonometric wavelets

In this section, we shall give a brief introduction on the trigonometric scaling and wavelet functions. More details can be found in [13].

For all $j \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, two scaling functions $\varphi^0_{j,0}(\theta)$ and $\varphi^1_{j,0}(\theta)$ are defined as:

$$\varphi^0_{j,0}(\theta) := \frac{1}{2^{2j+1}} \sum_{k=0}^{2^{j+1}-1} D_k(\theta), \tag{1}$$

$$\varphi^1_{j,0}(\theta) := \frac{1}{2^{2j+1}} \left(\widetilde{D}_{2^{j+1}-1}(\theta) + \frac{1}{2} \sin 2^{j+1}\theta \right), \tag{2}$$

where $D_n(\theta) := 1/2 + \sum_{k=1}^{n} \cos k\theta$, $\widetilde{D}_n(\theta) := \sum_{k=1}^{n} \sin k\theta$ denote the Dirichlet kernel and the conjugate kernel. For $n = 0,1,\ldots,2^{j+1}-1$, define $\varphi^i_{j,n}(\theta) = \varphi^i_{j,0}(\theta - \theta_{j,n}), i = 0,1$. Furthermore, for notational convenience, let $\varphi^i_{j,n} = \varphi^i_{j,n \bmod 2^{j+1}}, i = 0,1$ for any $n \in \mathbb{Z}$.

Then the following interpolatory properties hold for each k, $n = 0,1,\ldots,2^{j+1}-1$:

$$\varphi^0_{j,n}(\theta_{j,k}) = \delta_{kn}, \quad \varphi^{0\prime}_{j,n}(\theta_{j,k}) = 0, \quad \varphi^1_{j,n}(\theta_{j,k}) = 0, \quad \varphi^{0\prime}_{j,n}(\theta_{j,k}) = \delta_{kn}, \tag{3}$$

where $\theta_{j,k} = k\pi/2^j$ denotes the nodes for interpolation.

For all $j \in \mathbb{N}_0$, two wavelet functions $\psi^0_{j,0}(\theta), \psi^1_{j,0}(\theta)$ are defined as

$$\psi^0_{j,0}(\theta) = \frac{1}{2^{j+1}} \cos 2^{j+1}\theta + \frac{1}{3 \cdot 2^{2j+1}} \sum_{n=2^{j+1}+1}^{2^{j+2}-1} (3 \cdot 2^{j+1} - n) \cos n\theta, \tag{4}$$

$$\psi^1_{j,0}(\theta) = \frac{1}{3 \cdot 2^{2j+1}} \sum_{n=2^{j+1}+1}^{2^{j+2}-1} \sin n\theta + \frac{1}{2^{2j+3}} \sin 2^{j+2}\theta. \tag{5}$$

As for the scaling functions, for all $j \in \mathbb{N}_0$ and $n = 0,1,\ldots,2^{j+1}-1$, set $\psi^0_{j,n}(\theta) = \psi^0_{j,0}(\theta - \theta_{j,n})$ and $\psi^1_{j,n}(\theta) = \psi^1_{j,0}(\theta - \theta_{j,n})$ with the same use of indices modulo 2^{j+1}. The wavelet functions also have the similar interpolatory properties as the scaling functions.

Now, we can define the scaling function spaces $V_j (j \in \mathbb{N}_0)$ and the wavelet function spaces $W_j (j \in \mathbb{N}_0)$ as