Applied Mathematics and Computation 218 (2012) 5580–5587

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Hierarchical least squares based iterative estimation algorithm for multivariable Box–Jenkins-like systems using the auxiliary model

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ARTICLE INFO

Keywords:
System modelling
Least squares
Parameter estimation
Auxiliary model identification
Hierarchical identification
Multivariable Box–Jenkins-like model

ABSTRACT

This paper presents a hierarchical least squares iterative algorithm to estimate the parameters of multivariable Box–Jenkins-like systems by combining the hierarchical identification principle and the auxiliary model identification idea. The key is to decompose a multivariable systems into two subsystems by using the hierarchical identification principle. As there exist the unmeasurable noise-free outputs and noise terms in the information vector, the solution is using the auxiliary model identification idea to replace the unmeasurable variables with the outputs of the auxiliary model and the estimated residuals. A numerical example is given to show the performance of the proposed algorithm.

1. Introduction

Parameter estimation is a basic method for system modelling and signal filtering [1–4]. The recursive or iterative algorithms can estimate the system parameters or find the solutions of matrix equations [5–13]. Recently, parameter estimation of multivariable systems has received much attention in system identification [14–18]. For example, Ding et al. presented a gradient based and a least squares based iterative estimation algorithms for multi-input multi-output systems [19]; Xiang et al. proposed a hierarchical least squares algorithm for single-input multiple-output systems [20]; Han and Ding studied the convergence of the multi-innovation stochastic gradient algorithm for multi-input multi-output systems [18]; Bao et al. presented a least squares based iterative parameter estimation algorithm for multivariable controlled autoregressive moving average systems [21]. Liu et al. studied the convergence of stochastic gradient estimation algorithm for multivariable ARX-like systems [22].

Furthermore, Wang presented a least squares-based recursive and iterative estimation for output error moving average (OEMA) systems using data filtering [23]; Wang and Ding derived an input–output data filtering based recursive least squares parameter estimation for CARARMA systems [24]; Ding et al. proposed a gradient based and a least-squares based iterative identification methods for OE and OEMA systems [25] and several recursive and iterative identification methods for Hammerstein systems [26]. Other identification methods can be found for Hammerstein OEMA systems, Hammerstein OEAR systems and Wiener systems in [27–34] and for linear regressive models [35–40].

Some novel identification methods are born often, e.g., the multi-innovation identification methods for linear and pseudo-linear regression models [18,41–51], the auxiliary model based identification methods and the hierarchical identification methods for dual-rate and non-uniformly sampled-data systems or missing-data systems [49,52–64].

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doi:10.1016/j.amc.2011.11.051

Hierarchical identification is based on the decomposition and can deal with parameter estimation for multivariable systems. Ding et al. proposed a hierarchical gradient iterative algorithm and a hierarchical least squares iterative algorithm for multivariable discrete-time systems [65,66]; Han et al. presented a hierarchical least-squares based iterative identification algorithm for multivariable CARMA-like model [67]; Zhang et al. studied the hierarchical gradient based iterative estimation algorithm for multivariable output error moving average systems (i.e., multivariable OEMA-like systems) [67,68]. On the basis of the work in [68], this paper considers the identification problem of general stochastic multivariable Box–Jenkins-like systems with an ARMA noise disturbance.

The paper is organized as follows. Section 2 describes the problem formulation related to the multivariable Box–Jenkins-like systems. Section 3 derives the hierarchical least squares iterative algorithm for the multivariable Box–Jenkins-like systems. Section 4 provides a numerical example to illustrate the proposed method. Finally, we offer some concluding remarks in Section 5.

2. System description

Recently, Han et al. presented a hierarchical least squares based iterative identification algorithm for multivariable CARMA-like systems with moving average noises [67]:

$$\alpha(z)\boldsymbol{y}(t) = \boldsymbol{Q}(z)\boldsymbol{u}(t) + D(z)\boldsymbol{v}(t).$$

Zhang et al. derived a hierarchical gradient based iterative parameter estimation algorithm for multivariable output error moving average systems (i.e., multivariable OEMA systems [68]):

$$\boldsymbol{y}(t) = \frac{\boldsymbol{Q}(z)}{\alpha(z)}\boldsymbol{u}(t) + D(z)\boldsymbol{v}(t).$$

On the basis of the work in [67,68], this paper considers the following multivariable Box–Jenkins-like system (i.e., multivariable BJ-like model) shown in Fig. 1,

$$\boldsymbol{y}(t) = \frac{\boldsymbol{Q}(z)}{\alpha(z)}\boldsymbol{u}(t) + \frac{D(z)}{C(z)}\boldsymbol{v}(t), \tag{1}$$

where $\boldsymbol{y}(t) \in \mathbb{R}^m$ is the system output vector, $\boldsymbol{u}(t) \in \mathbb{R}^r$ is the system input vector, $\boldsymbol{v}(t) \in \mathbb{R}^m$ is a stochastic white noise vector with zero mean and variance σ^2, $\alpha(z)$ is a monic polynomial in the unit backward shift operator z^{-1} $[z^{-1}y(t) = y(t-1)]$, $\boldsymbol{Q}(z)$ is a matrix polynomial in z^{-1}, $C(z)$ and $D(z)$ is a polynomial in z^{-1}, and are defined by

$$\begin{aligned}
\alpha(z) &:= 1 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \cdots + \alpha_n z^{-n}, \quad \alpha_i \in \mathbb{R}^1,\\
\boldsymbol{Q}(z) &:= \boldsymbol{Q}_1 z^{-1} + \boldsymbol{Q}_2 z^{-2} + \cdots + \boldsymbol{Q}_n z^{-n}, \quad \boldsymbol{Q}_i \in \mathbb{R}^{m\times r},\\
C(z) &:= 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_{n_c} z^{-n_c}, \quad c_i \in \mathbb{R}^1,\\
D(z) &:= 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_{n_d} z^{-n_d}, \quad d_i \in \mathbb{R}^1.
\end{aligned}$$

The objective of this paper is to derive a hierarchical least squares based iterative estimation algorithm to identify the parameters or parameter matrix (α_i, $\boldsymbol{Q}_i$, c_i, d_i) from given input–output data $\{\boldsymbol{u}(t),\boldsymbol{y}(t)\colon t = 1, 2, \ldots\}$.

3. The hierarchical least squares based iterative algorithm

Let

$$\boldsymbol{x}(t) := \frac{\boldsymbol{Q}(z)}{\alpha(z)}\boldsymbol{u}(t) \in \mathbb{R}^m, \tag{2}$$

$$\boldsymbol{w}(t) := \frac{D(z)}{C(z)}\boldsymbol{v}(t) \in \mathbb{R}^m. \tag{3}$$

Substituting (2) and (3) into (1) gives

$$\boldsymbol{y}(t) = \boldsymbol{x}(t) + \boldsymbol{w}(t). \tag{4}$$

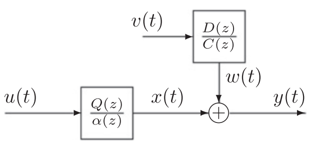

Fig. 1. The multivariable systems based on Box–Jenkins-like model.