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Resolution, in L^p-spaces, of transmission problems set in an unbounded domains

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ABSTRACT

The goal of this work is to give a complete study of some abstract transmission problems (P^δ), for every $\delta > 0$, set in unbounded domain composed of a half-line $]-\infty, 0[$ and a thin layer $]0, \delta[$. Existence and uniqueness results are obtained for strict solutions in UMD Banach spaces, by using essentially the semigroup theory and the Dore-Venni's Theorem given in [8].

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1. Introduction and motivation

Let us consider the following abstract transmission problem

$$(P^\delta)\begin{cases} (u_-^\delta)''(x) + Au_-^\delta(x) = g_-(x) \text{ on } (-\infty, 0) \\ (u_+^\delta)''(x) + Au_+^\delta(x) = g_+^\delta(x) \text{ on } (0, \delta) \\ u_-^\delta(0) = u_+^\delta(0), \mu_-(u_-^\delta)'(0) = \mu_+(u_+^\delta)'(0), \\ (u_+^\delta)'(\delta) = f_+^\delta. \end{cases}$$

where A is a closed linear operator in a complex Banach space E with domain $D(A)$, δ is a small positive fixed parameter in $]0, 1]$, the second members

$$g_- \in L^p(-\infty, 0; E), \quad g_+^\delta \in L^p(0, \delta; E),$$

with $1 < p < +\infty$, f_+^δ is some given element in E and μ_-, μ_+ are the conductibility positive coefficients of two bodies $(-\infty, 0)$ and $(0, \delta)$, depending possibly on δ.

We assume in all this work that

$$E \text{ is a UMD space,} \tag{1}$$

(we recall that a Banach space E is a UMD space if and only if for some $p > 1$ the Hilbert transform is continuous from $L^p(\mathbb{R}; E)$ into itself, see Bourgain [5] and Burkholder [6]). In concrete applications, E can be any $L^q(\Omega)$ ($1 < q < \infty$, Ω an open set of $\mathbb{R}^n$) or any interpolation space built on $L^q(\Omega)$; see our model example in the last section.

We suppose that

$$\begin{cases} \rho(A) \supset [0, +\infty[\quad \text{and} \quad \exists C > 0 \\ \forall \lambda \in [0, +\infty[: \|(A - \lambda I)^{-1}\| \leqslant \frac{C}{1+|\lambda|}, \end{cases} \tag{2}$$

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($\rho(A)$ denotes the resolvent set of A; this assumption means exactly the ellipticity of our problem as in Krein [14]), and

$$\begin{cases} \forall s \in \mathbb{R}, (-A)^{is} \in L(E) \quad \text{and} \\ \exists C > 1, \quad \alpha \in]0,\pi[: \forall s \in \mathbb{R}, \ \|(-A)^{is}\| \leqslant Ce^{\alpha|s|}. \end{cases} \tag{3}$$

(For the complex powers of operators, see, for instance, Haase [13].)

Remark 1.

1. From (1) and (2) we deduce that $D(A)$ is dense in E (see Haase [13], Proposition 2.1.1, pp. 18–19).
2. From (2) we deduce that, there exists $\theta_0 \in]0,\pi/2[$ and $r_0 > 0$ such that

$$\rho(A) \supset S_{\theta_0,r_0} = \{z \in \mathbb{C}^* : |\arg z| \leqslant \theta_0\} \cup \overline{B(0,r_0)},$$

and the estimate in (2) remains true in S_{θ_0,r_0}.

3. Assumption (2) implies that $B := -(-A)^{1/2}$ generates an analytic semigroup

$$e^{tB}, \quad t > 0,$$

(see Balakhrishnan [1] p. 419). On the other hand, there exist two positive constants a, M such that, for all $t > 0$ we have

$$\|e^{tB}\|_E \leqslant Me^{-at},$$

and

$$\|B^m e^{tB}\|_E \leqslant Mt^{-m}e^{-at},$$

for every $m \in \mathbb{N}^*$ (see Pazy [19], Theorem 6.13, p. 74).

4. Assumption (3) implies that

$$\exists C > 1, \quad \alpha \in]0,\pi[: \forall s \in \mathbb{R}, \quad \left\| \left(\sqrt{-A}\right)^{is} \right\| \leqslant Ce^{\frac{\alpha}{2}|s|},$$

(see Haase [13], Proposition 3.2.1, pp. 62–63).

We will focus ourselves, in this first work, on the existence and uniqueness for the strict solution of (P^δ), that is, a function

$$u^\delta = \begin{cases} u_-^\delta \ \text{in }]-\infty,0[\\ u_+^\delta \ \text{in }]0,\delta[, \end{cases}$$

such that

$$\begin{cases} u_-^\delta \in W^{2,p}(-\infty,0;E) \cap L^p(-\infty,0;D(A)), \\ u_+^\delta \in W^{2,p}(0,\delta;E) \cap L^p(0,\delta;D(A)), \end{cases}$$

and verifying (P^δ). The main result of our work is summarized by the following.

Theorem 2. *Assume (1)–(3). Let*

$$g_+^\delta \in L^p(0,\delta;E), g_- \in L^p(-\infty,0;E),$$

with $1 < p < +\infty$. Then problem (P^δ) has a unique strict solution if and only if

$$f_+^\delta \in (D(A),E)_{\frac{1}{2p}+\frac{1}{2},p}.$$

We recall that for all $\alpha \in]0,1[$ and $q \in [1,\infty]$

$$(E,D(A))_{\alpha,q} = (D(A),E)_{1-\alpha,q}$$

is a real interpolation space between $D(A)$ and E (see Triebel [20] p. 96).

Our techniques to obtain this result are essentially based on the Dunford operational calculus, the analytic semigroup theory and the celebrated Dore-Venni's Theorem.

Several authors were interested in the study of transmission problems in different spaces with different boundary conditions in Hilbertian spaces, see [7,18]. In [10], Favini et al. have considered some transmission problems set on a bounded domain for a second member in an interpolation space. They used the Da Prato Grisvard sum theory of linear operators. In [2], Belhamiti et al. have studied the same problem (but set in bounded domains) in the framework of Hölder continuous spaces. They have obtained necessary and sufficient conditions on the data in order to prove optimal result of the strict solution. In our case, the second member is only in L^p spaces and the domain is unbounded.

The paper is organized as follows.

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