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Propagation of torsional surface waves in a homogeneous layer of finite thickness over an initially stressed heterogeneous half-space

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ABSTRACT

In the present paper, the dispersion equation which determines the velocity of torsional surface waves in a homogeneous layer of finite thickness over an initially stressed heterogeneous half-space has been obtained. The dispersion equation obtained is in agreement with the classical result of Love wave when the initial stresses and inhomogeneity parameters are neglected. Numerical results analyzing the dispersion equation are discussed and presented graphically. The result shows that the initial stresses have a pronounced influence on the propagation of torsional surface waves. It has also been shown that the effect of density, directional rigidities and non-homogeneity parameter on the propagation of torsional surface waves is prominent.

1. Introduction

There has been considerable interest generated in recent years on wave propagation through naturally occurring media and man-made materials in view of widespread applications in acoustic signal transmissions, seismically induced motions, noise control, non-destructive evaluation, subsurface exploration etc. [1–3]. The study of surface waves in a half-space is important to seismologists due to its possible applications in Geophysical prospecting and in understanding the cause and estimation of damage due to earthquakes. Surface waves carry the greatest amount of energy from shallow shocks and are of primary cause of destruction that can result from earthquakes. Haskell [4] studied the dispersion of surface waves in multilayered media. A source on elastic waves is the monograph by Ewing et al. [5]. In a classical work, Jones [6] pointed out some possible practical applications in which the velocity of propagation of waves in layered media is computed to view the problem. The history of the research of surface waves in elastic media has its beginning in the paper by Lord Rayleigh in 1885 [7]. Wave propagation and profile reconstruction in inhomogeneous elastic media has been studied by Bagnas [8]. Surface waves take different forms and exist in a broad frequency range governing more than 10 orders of magnitude. Surface waves are studied primarily within the scope of single-component models [9–11]. At the present time, little work has been done in the analysis of surface waves propagating through multiphase media, in particular when there are two constituents (solid and fluid).

One type of surface wave may be available in non-homogeneous Earth known as torsional surface waves (i.e. waves with amplitudes exponentially decaying with distance from the free surface). These waves are horizontally polarized but give a twist to the medium when it propagates. Although much information is available on the propagation of surface waves such as Rayleigh waves, Love waves and Stonely waves etc., the torsional wave has not drawn much attention and very little literature is available on the propagation of this wave. The Earth is considered to be a layered elastic medium with variation in density and rigidity in constituent layers. Therefore the torsional surface wave must propagate during earthquakes.The near

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surface of the Earth consists of layers of different types of material properties overlying a half space of various types of rock, underground water, oil and gases. So, the studies of the propagation of torsional surface waves will be of great interest to seismologists.

Some papers have been published on the propagation of torsional waves in elastic medium with different types of inhomogeneity. Rayleigh [7] in his remarkable paper showed that the isotropic homogeneous elastic half-space does not allow a torsional surface wave to propagate, in this connection Georgiadis et al. [12] have examined the torsional surface wave in a gradient-elastic half-space. Meissner [13] pointed out that in an inhomogeneous elastic half-space with quadratic variation of shear modulus and density varying linearly with depth, torsional surface waves do exist. Vardoulakis [14] has studied the problem on torsional surface waves in inhomogeneous elastic media. Dey et al. [15] studied the propagation of torsional waves in a homogeneous substratum over a heterogeneous half-space. Gupta et al. [16] have studied effect of irregularity on the propagation of torsional surface waves in an initially stressed anisotropic poro-elastic layer. Torsional wave dispersion relations in a pre-stressed bi-material compounded cylinder with an imperfect interface have been studied by Kepceler [17]. Ozturk and Akbbarov [18] have studied torsional wave propagation in a pre-stressed circular cylinder embedded in a pre-stressed elastic medium. Murakami and Yamakawa [19] studied torsional wave propagation in reinforced concrete columns. Akbarov et al. [20] have studied torsional wave dispersion in a finitely pre-strained hollow sandwich circular cylinder. Also torsional waves in a stressed elastic tube filled with a viscous fluid has been studied by Demiray [21].

Propagation of torsional surface waves in heterogeneous half-space with irregular free surface has been studied by Selim [22]. He has discussed the effect of irregularity and heterogeneity on the propagation of torsional surface waves. The propagation of torsional waves in a prestressed compound (bi-layered) hollow circular cylinder has been investigated by Ozturk and Akbarov [23] within the framework of a piecewise homogeneous body model, with the use of a three-dimensional linerized theory of elastic waves in initially stressed bodies. Torsional wave dispersion relations in a pre-stressed bi-material compounded cylinder has been studied by Ozturk and Akbarov [24].

Propagation of torsional surface wave in anisotropic poroelastic medium under initial stress has been studied by Chattaraj et al. [25] in which the possibility of propagation of torsional surface wave in fluid saturated poroelastic layer lying over non-homogeneous elastic half-space has been discussed. They have considered hyperbolic and quadratic types of inhomogeneity. The inhomogeneity factor due to quadratic and hyperbolic variations in rigidity, density and initial stress of the medium decreases the phase velocity as it increases.

Chattopadhyay et al. [26] have studied propagation of torsional waves in an inhomogeneous layer over an inhomogeneous half-space which is concerned with the study of propagation of torsional waves in an inhomogeneous isotropic layer whose material properties vary harmonically with a space variable, lying over a semi-infinite inhomogeneous isotropic half-space.

The development of initial stresses in the medium is due to many reasons, for example resulting from the difference of temperature, process of quenching, shot peening and cold working, slow process of creep, differential external forces, gravity variations etc. These stresses have a pronounced influence on the propagation of waves as shown by Biot [27]. The Earth is also an initially stressed medium. It is therefore of much interest to study the influence of these stresses on the propagation of torsional surface wave. The study of surface waves in an initially stressed medium is of interest not for theoretical taste only but for practical purposes too. Based on the pioneering work of Biot [27] on pre-stressed solids, various studies of body and surface wave propagation in the pre-stressed solids have been carried out by many researchers such as Chattopadhyay et al. [28], Kar and Kalyani [29], Dey and Addy [30] and Roy [31].

In the present paper torsional surface wave propagation in a homogeneous layer of finite thickness over an initially stressed heterogeneous half-space has been studied. The inhomogeneity of the half-space has been taken as

$$N = N_1(1+\alpha z), \quad L = L_1(1+\beta z), \quad P = P_1(1+\gamma z) \quad \text{and } \rho = \rho_1(1+\delta z)$$

where N, L are directional rigidities, ρ is the density, P is the compressive initial stress at any point in the layer which is assumed to be transversely isotropic with z-axis as the axis of symmetry and α is a constant having dimension that is inverse of length. The inhomogeneity of the half-space has been taken along the z-direction.

In order to show the effect of initial stresses, density, rigidity and non-homogeneity on the propagation of torsional surface waves, numerical computation of dispersion equation have been performed with different values of parameter representing the above characteristics. It has been found that as compressive initial stresses increases the velocity of torsional surface wave increases for the same frequency. It has been observed that decrease in tensile initial stress decreases the velocity of torsional surface wave for the same frequency. It is seen that as the value of density parameter increases, the velocity of torsional surface wave decreases. It has also been found that as the value of directional rigidity parameters increase, the velocity of torsional surface wave decreases.

2. Formulation

Consider a homogeneous layer of finite thickness H, over a vertically heterogeneous initially stressed half-space. The heterogeneity has been considered in rigidity, density and initial stress. The origin of the cylindrical co-ordinate system (r, θ, z) is located at the interface separating the layer from the half-space, and the z-axis is directed downwards (as shown in Fig. 1). The homogeneity of the layer has been taken as $\mu = \mu_0$ and $\rho = \rho_0$ where μ and ρ are the rigidity and mass density of the