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Study on the behavior of oscillating solitons using the (2+1)-dimensional nonlinear system

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ABSTRACT

By means of an extended homogeneous balance method and a variable separation hypothesis, a broad general variable separation solution with three specific arbitrary functions of the nonlinear (2+1)-dimensional Broer-Kaup (BK) equations was derived. Based on the derived solution, a number of abundant oscillating solitons, such as dromion, multi-dromion, solitoff, ring, multi-lump and so on, have been revealed in this study by selecting appropriate functions of the general variable separation solution.

1. Introduction

Nonlinear partial differential equations are widely used to study and describe complex phenomena in various fields of natural sciences, such as biology, chemistry, mathematics, communication and, in particularly, almost all areas of physics. The applications in physics can be highlighted to include condensed matter physics, field theory, fluid dynamics, plasma physics and optics. It is our understanding that searching for exact and explicit solutions of a nonlinear physical model, especially for new exponentially localized structures (e.g. soliton solutions or these excitations with novel properties) has been a very interesting work in many laboratories. In the literatures published recently, several significant (2+1)-dimensional models were investigated and some special properties of localized solutions for those models were revealed by means of different approaches [1–8]. From those studies, one can see that there are more abundant localized structures in higher dimensions than lower dimensions. It is believed that many localized structures may still not be revealed using the currently available approaches. For instance, oscillation is a ubiquitous phenomenon in natural world. Therefore, the localized excitations in high-dimensional systems ought to possess specific oscillating properties which deserve more attentions in future research Furthermore, in some degree, some oscillating solitons may be considered as one kind of nonpropagation solitons [6,7]. Motivated by these reasons, we intended to take the (2+1)-dimensional Broer-Kaup (BK) equations:

$$H_{ty} = H_{xxy} - 2(HH_x)_y - 2G_{xx}, \tag{1a}$$

$$G_t = -G_{xx} - 2(GH)_x, \tag{1b}$$

which were obtained from a Kadomtsev-Petviashvili (KP) equation by the symmetry constraints [9], as a concrete example to study some possible oscillating soliton structures in higher-dimensional physical models. Starting from a special Backlund transformation obtained by using extended homogenous balance method and the variable separation hypothesis, we converted the BK system into a relatively simple variable separation equation, and then obtained a broad general solution. Some types of the usual localized excitations of Eq. (1) such as multi-dromion, multi-lump and multi-ring soliton, can be easily constructed by selecting the appropriate arbitrary functions[4]. In addition to the usual localized structures, some new

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localized excitations, e.g. fractal dromion, fractal lump, and multiple peakon excitations, have been found by selecting some types of lower-dimensional appropriate functions [10].

This paper is organized as follows. In Section 2, we apply the variable separation approaches based on the extended homogeneous balance method (VSAonEHBM) to the BK system and obtain its exact excitations. In Section 3, we give some selected oscillating solitons, such as dromion, multi-dromion, solitoff, ring, multi-lump and so on, by selecting appropriate functions in the general variable separation solution of the BK system to demonstrate some interesting outcomes. A brief summary and discussions are made in the last section.

2. Variable separation approaches based on the extended homogeneous balance method for the (2+1)-dimensional BK system

According to the EHBM, let

$$H = f'(w)w_x + a(x,t), \tag{2a}$$

$$G = g''(w)w_x w_y + g'(w)w_{xy}, \tag{2b}$$

where the prime denotes d/dw, $f(w)$, $g(w)$ and $a(x,t)$ are undetermined functions. Substituting Eq. (2) into Eq. (1), we can obtain

$$H_{ty} - H_{xxy} + 2(HH_x)_y + 2G_{xx} = (2f''^2 + 2f'f^{(3)} - f^{(4)} + 2g^{(4)})w_y w_x^3 + \text{lower power terms of the derivatives of } w(x,y,t) \text{ respect to } x, y \text{ and } t. \tag{3a}$$

$$G_t + G_{xx} + 2(GH)_x = (g^{(4)} + 2f'g^{(3)} + 2g''f'')w_y w_x^3 + \text{lower power terms of the derivatives of } w(x,y,t) \text{ respect to } x, y \text{ and } t. \tag{3b}$$

Setting the coefficients of $w_y w_x^3$ in Eq. (3) to zero, it yields an ordinary differential system

$$2f''^2 + 2f'f^{(3)} - f^{(4)} + 2g^{(4)} = 0, \tag{4a}$$

$$g^{(4)} + 2f'g^{(3)} + 2g''f'' = 0. \tag{4b}$$

The following special solutions exist for Eq. (4):

$$f(w) = g(w) = \ln(w). \tag{5}$$

Thereby $f'f'' = -\frac{1}{2}f^{(3)}, f'^2 = -f^{(2)}$. Using these results, the expression Eq. (3) can be simplified as

$$H_{ty} - H_{xxy} + 2(HH_x)_y + 2G_{xx} = f'(w_t + w_{xx} + 2aw_x)_{xy} + f''[w_{xy}(w_t + w_{xx} + 2aw_x) + w_x(w_t + w_{xx} + 2aw_x)_y + w_y(w_t + w_{xx} + 2aw_x)_x] + f^{(3)}[w_x w_y(w_t + w_{xx} + 2aw_x)] = 0, \tag{6a}$$

$$G_t + G_{xx} + 2(GH)_x = g'(w_t + w_{xx} + 2aw_x)_{xy} + g''[w_{xy}(w_t + w_{xx} + 2aw_x) + w_x(w_t + w_{xx} + 2aw_x)_y + w_y(w_t + w_{xx} + 2aw_x)_x] + g^{(3)}[w_x w_y(w_t + w_{xx} + 2aw_x)] = 0. \tag{6b}$$

Setting the coefficients of $f^{(3)}, f'', f'$ and $g^{(3)}, g'', g'$ in Eq. (6) to zero and simplifying, it then yields a set of partial differential equations for $w(x,y,t)$:

$$(w_t + w_{xx} + 2aw_x)_{xy} = 0, \tag{7a}$$

$$w_x w_y(w_t + w_{xx} + 2aw_x) = 0, \tag{7b}$$

$$w_{xy}(w_t + w_{xx} + 2aw_x) + w_x(w_t + w_{xx} + 2aw_x)_y + w_y(w_t + w_{xx} + 2aw_x)_x = 0, \tag{7c}$$

$$(w_t + w_{xx} + 2aw_x)_{xy} = 0. \tag{7d}$$

By analyzing the above equations, we find that Eq. (7a)–(7d) are satisfied automatically under the conditions of

$$w_t + w_{xx} + 2aw_x = 0. \tag{8}$$

Considering the linear Eq. (8) of the original system, we can construct many types of special solutions. Because $a(x,t)$ is undetermined functions of $\{x,t\}$, we can select an appropriate variable separation hypothesis for function w as follows

$$w(x,y,t) = a_0 + pqs, \quad a(x,t) = -\frac{ps_t + p_{xx}s}{2p_x s}, \tag{9}$$

where a_0 is an arbitrary constant, and $p \equiv p(x)$, $q \equiv q(y)$, $s \equiv s(t)$ are three arbitrary variable separation functions of x, y and t, respectively. Substituting Eq. (5) and (9) into Eq. (2), we can obtain