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Optimal price and lot size determination for a perishable product under conditions of finite production, partial backordering and lost sale

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ABSTRACT

The paper describes an EOQ model of a perishable product for the case of price dependent demand, partial backordering which depends on the length of the waiting time for the next replenishment, and lost sale. The model is solved analytically to obtain the optimal price and size of the replenishment. In the model, the customers are viewed to be impatient and a fraction of the demand is backlogged. This fraction is a function of the waiting time of the customers. In most of the inventory models developed so far, researchers considered that inventory accumulates at the early stage of the inventory and then shortage occurs. This type of inventory is called IFS (inventory followed by shortage) policy. In the present model we consider that shortage occurs before the starting of inventory. We have proved numerically that instead of taking IFS, if we consider SFI (shortage followed by inventory) policy, we would get better result, i.e., a higher profit. The model is extended to the case of non-perishable product also. The optimal solution of the model is illustrated with the help of a numerical example.

1. Introduction

For many items, reduced price can increase its sell. So demand can be dependent on unit price of items. Therefore when the demand for the product is price sensitive, pricing and lotsizing decisions are very important and for the case of perishable product, the vendor may need to backlog demand to avoid costs due to deterioration. So perishability and backlogging are complementary conditions. Many inventory modelers developed their inventory models considering backlogging rate to be linearly dependent on the total number of customers in the waiting line. However the backlogging rate should depend on the time-spend in waiting for the arrival of the next lot. Montgomery et al. [1], Park [2], Mak [3], Chang and Dye [4], Abad [5,6], Ghosh and Chaudhuri [7], etc. presented inventory models considering backorders and lost sales.

In the present paper we assume partial backordering that depends on the waiting time for backlogging, which can better depict the customers unwillingness to wait for backlogging i.e., the backlogging rate decreases quickly in terms of waiting time.

Many inventory modelers developed their inventory models for perishable product under condition of finite production, fixed demand and backordering. Raafat [8] developed a model on continuously deteriorating items in an excellent way. Researcher like Rubin et al. [9], Das [10], Monahan [11], Tersine and Toelle [12], Kim and Hwang [13], Hwang et al. [14], Burwell [15], etc. developed their models considering quantity discount. Retailers optimal policy for a perishable product with shortages was considered by Giri et al. [16], when suppliers offers all-unit quantity and freight cost discounts. Jalan and Chaudhuri [17] studied a multi-product EOQ model for deteriorating items with pricing consideration and inventory

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shortages. A joint pricing ordering policy for deteriorating inventory was developed by Mukhopadhyay et al. [18]. Optimal selling price and lot size with time varying deterioration with partial backlogging was discussed by Sana [19]. On the other hand, optimal ordering and pricing policy with order cancelation was discussed by You and Wu [20]. Multiproduct production quantity model with repair failure and partial backordering was discussed by Taleizadeh et al. [21].

Most of the above researchers formulate the problem as a cost minimization problem and uses backorders and lost sale costs. Abad [5] has proved the existence of a global minimum of the cost minimization problem in an excellent way. Goyal and Gunasekaran [22] have shown how pricing and advertising affect the demand in their model to determine the lot-sizes that maximize the total net profit. Luo [23] extended the above model by allowing complete backlogging of demand. He uses an enumeration procedure for finding the optimal lot size and maximum backorder level. Abad [6] considered optimal pricing and lot-sizing of a perishable product under conditions of finite production, partial backordering and lost sales. We call the model of Abad [6] as an IFS (inventory followed by shortages) policy. The SFI (shortage followed by inventory) policy is followed in the present paper. We consider that the inventory shortage occurs first in the model of Abad [6]. We shall show that the inventory policy suggested by Goyal et al. [24], allowing shortages in the beginning of the inventory cycle, will lead a better result, i.e., a higher profit.

2. Assumptions and notations

The following assumptions and notations have been used in developing the model:

1 A single-item inventory is considered over an infinite planning horizon.
2 The production rate for the item (units/period) is R.
3 Selling price per unit within the inventory cycle is p.
4 Demand rate (units/period) is a function of price p and is denoted by $D(p)$ and $R > D(p)$. There is no repair or replacement of the deteriorated items.
5 The items deteriorate continuously at a rate σ (constant), $0 < \sigma < 1$.
6 There is no repair or replacement of the deteriorated items during the production cycle.
7 Lead time is assumed to be zero.
8 Shortages are allowed.
9 The customers are assumed to be impatient and only a fraction $B(\eta)$ of the demand during the stock-out period is backlogged where η is the amount of time for which the customer waits before receiving goods and remaining fraction $1 - B(\eta)$ is lost. B (η) is given by $B(\eta) = \frac{k_0}{1+k_1\eta},\ 0 < k_0 < 1,\ 0 < k_1 < 1$. This is practical because as the waiting time of customers for receiving their goods increases, the backlogging rate decreases. Many inventory modelers developed their inventory models considering backlogging rate to be B $(\eta) = k_0 e^{-k_1\eta},\ 0 < k_0 < 1,\ 0 < k_1 < 1$. But in real life situation backlogging rate should not be as high as exponential. Therefore taking backlogging rate as $B(\eta) = \frac{k_0}{1+k_1\eta},\ 0 < k_0 < 1,\ 0 < k_1 < 1$ seems to be better representation than that of exponential waiting time dependent rate.
10 The inventory level at any time t is $I(t)$.
11 Duration of an inventory cycle when stockout condition exists is T.
12 Duration of an inventory cycle when there is positive inventory is λ.
13 β, ψ = interim time-span.
14 $D' = \frac{dD(p)}{dp} < 0$ for all $p \in (0,\infty)$.
15 The marginal revenue $\frac{d\{pD(p)\}}{dD} = p + \frac{D(p)}{D'}$ is an strictly increasing function of p and thus $\frac{1}{D(p)}$ is convex function of p.

3. Formulation and solution of the model

A typical behavior of the production-inventory in a cycle is depicted in Fig. 1. The inventory starts with zero stock at zero time. So shortage begins to accumulate at the early stage in inventory cycle. The production starts at time β to meet the current and backlogged demand. T is the time when the shortage level reaches to zero; afterwards a positive level of inventory begins to build up. At time $T + \psi$, the production process stops; the inventory level then stars declining. Finally the cycle ends with zero stock at time $T + \lambda$.

The instantaneous inventory level $I(t)$ at any time $t(0 \leqslant t \leqslant T + \lambda$ can be described by the following differential equations:

$$\frac{dI(t)}{dt} = -DB(\beta - t), \quad 0 \leqslant t \leqslant \beta, \quad I(0) = 0, \tag{1}$$

$$\frac{dI(t)}{dt} = R - D, \quad \beta \leqslant t \leqslant T, \quad I(T) = 0, \tag{2}$$

$$\frac{dI(t)}{dt} + \sigma I(t) = R - D, \quad T \leqslant t \leqslant T + \psi, \quad I(T) = 0, \tag{3}$$

$$\frac{dI(t)}{dt} + \sigma I(t) = -D, \quad T + \psi \leqslant t \leqslant T + \lambda, \quad I(T + \lambda) = 0. \tag{4}$$