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Generations and mechanisms of multi-stripe chaotic attractors of fractional order dynamic system

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ABSTRACT

In this paper, the generations of multi-stripe chaotic attractors of fractional order system are considered. The original fractional order chaotic attractors can be turned into a pattern with multiple "parallel" or " rectangular" stripes by employing certain simple periodic nonlinear functions. The relationships between the parameters relate to the periodic functions and the shape of the generated attractors are analyzed. Theoretical investigations about the underlying mechanisms of the parallel striped attractors of fractional order system are presented, with the fractional order Lorenz, Rössler and Chua's systems as examples. Moreover, the periodic doubling striped route to chaos of fractional order Rössler system and maximum Lyaponov exponent calculations are also given.

1. Introduction

In recent years chaotic behaviors and complex patterns generated from simple nonlinear dynamics have been extensively studied from different points of views, attracting the interests of researchers from various fields including particularly physics, biology, and engineering [1,6,9]. The studies of Chaos, including their butterfly-shapes [5], triple-well [14], multi-scrolls [21], and spherical patterns of chaotic attractors [2] etc., have benefited the investigation of the intrinsic nonlinear structures, complex behaviors of natural system as well as man-made systems. Various chaotic patterns have been used actively in practical applications such as complex pattern formation and secure communications [10,12].

The generation of complex (dynamic) patterns and phenomena is related to both the interaction between the systems and their respective attractors. In [3], chaotic attractors with multiple "parallel" stripes and specific stripe induced intermittent phenomena were studied, and multi-stripe chaotic attractors were produced on the basis of the special structure of the Rössler system. Then continuous systems and even smooth systems were also found to be able to generate similar phenomena in [4]. In these works, the "rectangular" striped attractor were produced by adding some periodic terms to $\dot{x}$ and $\dot{y}$ equations of the Rössler system. Moreover,the periodic couplings of the Rössler system could also generate multiple stripes [17]. In [18], the authors found the general method of multi-stripe chaotic attractors generations by employing some simple nonlinear periodic functions for chaotic of ordinary differential systems. However, for fractional order chaotic systems, whether the striped attractors can still occur by using the method proposed by [18] has not been discussed yet.

Fractional order differential equations have attracted increasing attention and have been applied in many ways in physics and engineering in recent decades [11,15,16]. Chaos can also be generated from them, such as the fractional order Rössler system [7], the fractional order Chua's system [22], the fractional order Chen system [8].

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In this paper, we discuss the generation of multi-stripe attractors from fractional order chaotic systems by using the method of [18]. The theoretical analysis is also given. Numerical simulations on various fractional order chaotic attractors verify the generality of the method.

The rest of the paper is organized as follows. The preliminaries about the fractional order dynamic systems are given in Section 2. The general method using periodic functions to produce fractional order attractors with multiple parallel stripes is testified in Section 3, and the theoretical results about the generation of "parallel" multi-stripe attractors from the fractional order dynamic system are given. In Section 4, multi-stripe attractors in rectangular forms are studied by the same technical line. Finally, some concluding remarks are given in Section 5.

2. Fractional calculus

In this section, we review the basic definition of the Riemann–Liouville fractional integrals and the Caputo's fractional derivatives and introduce our method of numerical simulations. Some propositions to be used in the following sections are also given.

Definition 2.1 (*see[11]*). Provided $\beta > 0$, the operator J^β defined on $\mathbb{L}_1[0,T]$ by

$$J^\beta y(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-\tau)^{\beta-1} y(\tau) d\tau \tag{1}$$

for $t \in [0,T]$, is called the Riemann–Liouville fractional integral operator of order β, where $\mathbb{L}_1[0,T] := \{y(t) : [0,T] \to \mathbb{R};\ y(t)$ is measurable on $[0,T]$ and $\int_0^T |y(t)| dt < \infty\}$ and $\Gamma(\cdot)$ is the Gamma function.

Definition 2.2 (*see[11]*). The operator $D^\alpha (n-1 < \alpha \leqslant n)$ defined by

$$D^\alpha y(t) = J^{n-\alpha} \frac{d^n}{dt^n} y(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^T (t-\tau)^{n-\alpha-1} \frac{d^n}{d\tau^n} y(\tau) d\tau \tag{2}$$

for $t \in [0,T]$ and $y(t) \in \mathbb{C}^n[0,T]$, is called the Caputo differential operator of order α.

In this paper, we simulate numerically the fractional order initial value problem

$$\begin{cases} D^\alpha X(t) = F(t, X(t)), t \in (0,T], \quad 0 < \alpha \leqslant 1, \\ X(0) = X^0, \end{cases} \tag{3}$$

with the following first order numerical method (see [20])

$$\widetilde{X}_j = h^\alpha F(t_j, \widetilde{X}_j) - \sum_{k=1}^{j} \omega_k \widetilde{X}_{j-k} - \left(\frac{1}{j^\alpha \Gamma(j-\alpha)} - \sum_{r=0}^{j} \omega_r \right) X^0 + \frac{h^\alpha}{t_j^\alpha} X^0, \quad j = 1, 2, \ldots, N, \tag{4}$$

where

$$X : [0,T] \mapsto \mathbb{R}^m, \quad F : [0,T] \times \mathbb{R}^m \mapsto \mathbb{R}^m, \quad F_i : [0,T] \times \mathbb{R}^m \mapsto \mathbb{R}, \quad X^0 = \left[X_1^0, X_2^0, \ldots, X_m^0\right]^T, \quad \widetilde{X}_j \approx X(t_j),$$
$$D^\alpha X(t) = \left[D^\alpha X_1(t), D^\alpha X_2(t), \ldots, D^\alpha X_m(t)\right]^T, \quad F(t,X) = [F_1(t,X), F_2(t,X), \cdots, F_m(t,X)]^T, \quad h = T/N, \quad t_j = jh,$$
$$j = 0, 1, 2, \ldots, N, \quad \omega_0 = 1, \quad \omega_k = \left(1 - \frac{\alpha+1}{k}\right) \omega_{k-1}, \quad k = 1, 2, \ldots, N.$$

The numerical method (4) is in accord with (5.10) in [20] when $m = 1$.

Proposition 2.1. *If $0 < \alpha \leqslant 1$, f is the operator from $\mathbb{C}^1[0,+\infty)$ to $\mathbb{C}^1(-\infty,+\infty)$, where $x \in \mathbb{C}^1[0,+\infty)$, then, $\exists \xi \in [0,t]$, s.t. $J^{1-\alpha}(f'(x)x') = f'(x(\xi)) D^\alpha x$.*

Proof. From the definition of the Caputo operator, we have

$$D^\alpha f(x) = J^{1-\alpha} \frac{d}{dt} f(x) = \frac{1}{\Gamma(1-\alpha)} \int_0^T (t-\tau)^{-\alpha} f'(x(\tau)) x'_\tau d\tau.$$

Applying the mean value theorem of integrals, then

$$D^\alpha f(x) = \frac{f'(x(\xi))}{\Gamma(1-\alpha)} \int_0^T (t-\tau)^{-\alpha} x'_\tau d\tau \ (\xi \in [0,t]) = f'(x(\xi)) D^\alpha x. \qquad \square$$