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Stability of Bidirectional Associative Memory networks with impulses ☆

Zhiguo Luo [a], Jianli Li [a,*], Jianhua Shen [b]

[a] Department of Mathematics, Hunan Normal University, Changsha, Hunan 410081, PR China
[b] Department of Mathematics, Hangzhou Normal University, Hangzhou, Zhejiang 310036, PR China

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ABSTRACT

By using Lyapunov functional and some analysis technique, sufficient conditions are obtained for the existence and asymptotic stability of a unique equilibrium of a Bidirectional Associative Memory (BAM) neutral network with Lipschitzian activation functions without assuming their boundedness, monotonicity or differentiability and subjected to impulsive state displacements at fixed instants of time. The sufficient conditions are in terms of the parameters of the network only and are easy to verify.

1. Introduction

In recent years, a class of neural network related to Bidirectional Associative Memory (BAM) has been proposed. These models generalize the single-layer auto-associative circuit. Therefore, this class of network possesses good application prospects in the area of pattern recognition, signal and image process etc. [1,2]. The research on stability of BAM network has many nice works [3–7]. The dynamical characteristics of the network are assumed to be governed by the dynamics of the following system of ordinary differential equations

$$\begin{aligned} \frac{dx_i(t)}{dt} &= -a_i x_i(t) + \sum_{j=1}^{p} c_{ij} f_j(y_j(t)) + c_i, t > t_0, \\ \frac{dy_j(t)}{dt} &= -b_j y_j(t) + \sum_{i=1}^{m} d_{ji} g_i(x_i(t)) + e_j, t > t_0, \end{aligned} \tag{1.1}$$

in which $i = 1,2,\ldots,m, j = 1,2,\ldots,p$; $x_i(t), y_j(t)$ denote the potential (or voltage) of the cell i and j at time t respectively, a_i, b_j are positive constants, they denote the rate with which the cell i and j reset their potential to the resting state when isolated from the other cells and inputs; the connection weights c_{ij}, d_{ji} are real numbers, they denote the strengths of connectivity between the cells j and i at time t respectively; c_i, e_j denote the ith and jth component of an external input source introduced from outside the network to the cell i and j respectively, the functions f_j, g_i: $R \to R$ represent the response of the jth and ith cell to its membrane potential and are known as activation functions; we denote $x(t) = (x_1(t),\ldots,x_m(t))^T \in R^m$, $y(t) = (y_1(t),\ldots,y_p(t))^T \in R^p$.

Dynamical systems are often classified into two categories of either continuous-time or discrete-time systems. Recently there has been a somewhat a new category of dynamical systems, which is neither purely continuous-time nor purely discrete-time ones; these are called dynamical systems with impulses. It is known that the theory of impulsive differential equations provides a natural framework for mathematical modeling of many real word phenomena. Significant progress has been made in the theory of impulsive differential equation in recent years [8–16,18].

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* Corresponding author.
E-mail address: ljianli@sina.com (J. Li).

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In this paper, inspired by Refs.[3,8,17,18], we consider the system (1.1) subjected to certain impulsive state displacements at fixed moments of time:

$$\left.\begin{array}{l} \frac{dx_i(t)}{dt} = -a_i x_i(t) + \sum\limits_{j=1}^{p} c_{ij} f_j(y_j(t)) + c_i, \quad t > t_0,\ t \neq t_k, \\ \frac{dy_j(t)}{dt} = -b_j y_j(t) + \sum\limits_{i=1}^{m} d_{ji} g_i(x_i(t)) + e_j, \quad t > t_0,\ t \neq t_k, \\ x(t_0^+) = x_0 \in R^m, \quad y(t_0^+) = y_0 \in R^p, \\ x_i(t_k^+) - x_i(t_k^-) = I_k(x_i(t_k^-)), \quad k = 1,2,\ldots, \\ y_j(t_k^+) - y_j(t_k^-) = \widetilde{I}_k(y_j(t_k^-)), \quad k = 1,2,\ldots, \\ t_0 < t_1 < t_2 < \cdots < t_k \to \infty \text{ as } k \to \infty, \end{array}\right\} \tag{1.2}$$

where $x_i(t_k^+) = x_i(t_k),\ y_j(t_k^+) = y_j(t_k)$, the functions $I_k(\cdot),\ \widetilde{I}_k(\cdot) : R \to R$ are assumed to be continuous.

The present paper is organized as follows. In Section 2, we obtain a set of sufficient conditions for the existence of a unique equilibrium of the system. We then study, in Section 3, the stability of this equilibrium solution. Finally we work out an example.

2. Existence of equilibria

In this section we consider network with globally Lipschitz activation functions without requiring them to be bounded, monotonic or differentiable. This is a significant advance in the area of BAM networks. Here we establish a number of easily verifiable sufficient conditions for the existence of unique equilibrium states. An equilibrium solution of (1.2) is a constant vector $x^* = (x_1^*, x_2^*, \ldots, x_m^*)^T \in R^m$ and $y^* = (y_1^*, y_2^*, \ldots, y_p^*)^T \in R^p$ which satisfy the system

$$\begin{aligned} a_i x_i^* &= \sum_{j=1}^{p} c_{ij} f_j(y_j^*) + c_i, \quad i = 1,\ldots,m, \\ b_j y_j^* &= \sum_{i=1}^{m} d_{ji} g_j(x_i^*) + e_j, \quad j = 1,\ldots,p, \end{aligned} \tag{2.1}$$

when the impulsive jumps $I_k(\cdot)$ and $\widetilde{I}_k(\cdot)$ are assumed to satisfy $I_k(x_i^*) = 0,\ \widetilde{I}_k(y_j^*) = 0,\ k = 1,2,\ldots$. In order to derive sufficient conditions for the existence and stability of equilibria on the coefficients and the activation functions in (1.2), we recall some elementary facts about the norms of vectors. If $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, then we have a choice of vectors norms in R^n, for instance $\|x\|_1$, $\|x\|_2$, $\|x\|_\infty$ are the commonly used norms where

$$\|x\|_1 = \sum_{i=1}^{n} |x_i|, \quad \|x\|_2 = \left\{\sum_{i=1}^{n} |x_i|^2\right\}^{1/2}, \quad \|x\|_\infty = \max_{1 \leqslant i \leqslant n} |x_i|.$$

We are now ready to consider the existence of equilibrium states of Eq. (2.1).

Lemma 2.1. *Let $a_i (i = 1,\ldots,m)$, $b_j (j = 1,\ldots,p)$ be positive numbers. Suppose that functions $f_j(\cdot)$, $g_i(\cdot)$: $R \to R$, $i = 1,\ldots,m$, $j = 1,\ldots,p$ satisfy*

$$\begin{aligned} |f_j(u_j) - f_j(v_j)| &\leqslant L_j |u_j - v_j|, \quad j = 1,\ldots,p, \\ |g_i(u_i) - g_i(v_i)| &\leqslant \bar{L}_i |u_i - v_i|, \quad i = 1,\ldots,m. \end{aligned} \tag{2.2}$$

Suppose further that $a_i,\ c_{ij},\ L_j,\ b_j,\ d_{ji},\ \bar{L}_i$ are such that

$$a_i > \bar{L}_i \sum_{j=1}^{p} |d_{ji}|, \quad b_j > L_j \sum_{i=1}^{m} |c_{ij}|. \tag{2.3}$$

Then there exists a unique solution of the system (2.1).

Proof. F: $R^{m+p} \to R^m$ defined by

$$F(x_1,\ldots,x_m,y_1,\ldots,y_p) = \begin{pmatrix} F_1(x_1,\ldots,x_m,y_1,\ldots,y_p) \\ \vdots \\ F_m(x_1,\ldots,x_m,y_1,\ldots,y_p) \end{pmatrix}$$

where

$$F_i(x_1,\ldots,x_m,y_1,\ldots,y_p) = \sum_{j=1}^{p} c_{ij} f_j\left(\frac{y_j}{b_j}\right) + c_i, \quad i = 1,2,\ldots,m.$$