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Exact solutions for the nonlinear Schrödinger equation with variable coefficients using the generalized extended tanh-function, the sine–cosine and the exp-function methods

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ABSTRACT

In this article we find the exact traveling wave solutions of the generalized nonlinear Schrödinger (GNLS) equation with variable coefficients using three methods via the generalized extended tanh-function method, the sine–cosine method and the exp-function method. The main objective of this article is to compare the efficiency of these methods by delivering the exact traveling wave solutions of the proposed nonlinear equation.

1. Introduction

In the nonlinear science, many important phenomena in various fields can be described by the nonlinear evolution equations (NLEES). Searching for exact soliton solutions of NLEEs plays an important and a significant role in the study on the dynamics of those phenomena. With the development of soliton theory, many powerful methods for obtaining the exact solutions of NLEES have been presented, such as the extended tanh-function method [1–5], the tanh-sech method [6–8], the sine–cosine method [9–11], the homogeneous balance method [12,13], the exp-function method [14–17], the Jacobi elliptic function method [18–21], the F-expansion method [22], the homotopy perturbation method [23,24], the variational iteration method [25], the inverse scattering transformation method [26], the Bäcklund transformation method [27], the Hirota bilinear method [28,29] and so on. To our knowledge, most of the aforementioned methods are related to constant coefficients models. Recently, much attention has been paid to the variable-coefficient nonlinear equations which can describe many nonlinear phenomena more realistically than their constant-coefficient ones.

In the present article, we discuss two important objectives. Firstly, we apply the generalized extended tanh-function method, the sine–cosine method and the exp-function method to find the exact solutions of the following generalized nonlinear Schrödinger (GNLS) equation with variable coefficients [30]:

$$iu_x + \frac{1}{2}\beta(x)u_{tt} + \alpha(x)u|u|^2 - i\gamma(x)u = 0, \tag{1.1}$$

where $u = u(x,t)$ is a real or complex valued function of x, t and $i = \sqrt{-1}$. Secondly, we are comparing the efficiency of these methods. The coefficients $\alpha(x)$, $\beta(x)$ and $\gamma(x)$ in Eq. (1.1) are functions of the indicated variable x, satisfying the condition $\alpha(x)\beta(x) \propto \gamma(x)^2$ in order that Eq. (1.1) has the given solutions presented in this paper.

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2. Description of the generalized extended tanh-function method

Suppose that a nonlinear evolution equation is given by

$$F(u, u_t, u_x, u_{tt}, u_{xx}, \ldots) = 0, \tag{2.1}$$

where $u = u(x,t)$ is an unknown function, F is a polynomial in u and its partial derivatives, in which the highest order derivatives and non-linear terms are involved. In the following, we give the main steps of this method:

Step 1. Using the generalized wave transformation

$$u(x,t) = u(\xi), \quad \xi = p(x)t + q(x), \tag{2.2}$$

where $p(x)$ and $q(x)$ are differentiable functions of x to be determined. Then, Eq. (2.1) is reduced to the following ODE:

$$Q(u, p(x)u'(\xi), [p'(x)t + q'(x)]u'(\xi), p(x)^2 u''(\xi), \ldots) = 0, \tag{2.3}$$

where Q is a polynomial in u and its total derivatives, while $'$ denotes the derivative with respect to the indicated variable.
Step 2. We suppose that Eq. (2.3) has the following formal solution:

$$u(\xi) = a_0 + \sum_{i=0}^{N} \left[a_k Y^k(\xi) + a_{-k} Y^{-k}(\xi) \right], \tag{2.4}$$

where N is a positive integer, and a_0, a_k, a_{-k} are constants, while $Y(\xi)$ is given by

$$Y(\xi) = \tanh(\xi). \tag{2.5}$$

The independent variable (2.5) leads to the following derivatives:

$$\begin{aligned} \frac{d}{d\xi} &= (1 - Y^2)\frac{d}{dY}, \\ \frac{d^2}{d\xi^2} &= (1 - Y^{2)}\left[-2Y\frac{d}{dY} + (1 - Y^2)\frac{d^2}{dY^2}\right], \\ \frac{d^3}{d\xi^3} &= (1 - Y^{2)}\left[(6Y^2 - 2)\frac{d}{dY} + -6Y(1 - Y^2)\frac{d^2}{dY^2} + (1 - Y^2)^2\frac{d^3}{dY^3}\right], \end{aligned} \tag{2.6}$$

and so on.
Step 3. Determine the positive integer N in (2.4) by balancing the highest order derivatives and nonlinear terms in Eq. (2.3).
Step 4. Substituting (2.4) along with (2.6) into (2.3) and equating the coefficients of $t^j Y^s(\xi)$ to zero, we get a system of algebraic equations.
Step 5. Solving these algebraic equations by *Maple* or *Mathematica*, we get the values of a_0, a_k, a_{-k}, $p(x)$ and $q(x)$.
Step 6. Substituting these values into (2.4) and (2.2), we can obtain the exact traveling wave solutions of Eq. (2.1).

3. On using the generalized extended tanh-function method to solve Eq. (1.1)

Assume that the solution of Eq. (1.1) can be written in the form

$$u(x,t) = \upsilon(x,t)\exp[i\theta(x,t)], \tag{3.1}$$

where $\upsilon(x,t)$ and $\theta(x,t)$ are amplitude and phase functions respectively. Substituting (3.1) into (1.1) and separating the real and imaginary parts, we obtain

$$-\upsilon\theta_x + \frac{1}{2}\beta(x)[\upsilon_{tt} - \upsilon(\theta_t)^2] + \alpha(x)\upsilon^3 = 0 \tag{3.2}$$

and

$$\upsilon_x + \frac{1}{2}\beta(x)[2\upsilon_t\theta_t + \upsilon\theta_{tt}] - \gamma(x)\upsilon = 0. \tag{3.3}$$

Balancing υ_{tt} and υ^3 in Eq. (3.2), we have $N = 1$. We assume that Eqs. (3.2) and (3.3) have the following formal solutions

$$\upsilon(\xi) = a_0 + a_1 Y(\xi) + a_{-1} Y^{-1}(\xi), \tag{3.4}$$

$$\theta(x,t) = f(x)t^2 + g(x)t + h(x). \tag{3.5}$$