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Natural convection boundary layer flow of fluid with temperature-dependent viscosity from a horizontal elliptical cylinder with constant surface heat flux

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ABSTRACT

This study deals with the temperature-dependent viscosity effects on the natural convection boundary layer on a horizontal elliptical cylinder with constant surface heat flux. The mathematical problem is reduced to a pair of coupled partial differential equations for the temperature and the stream function, and the resulting nonlinear equations are solved numerically by cubic spline collocation method. Results for the heat transfer characteristics are presented as functions of eccentric angle for various values of viscosity variation parameters, Prandtl numbers and aspect ratios. Results show that an increase in the viscosity variation parameter tends to accelerate the fluid flow near the surface and increase the maximum velocity, thus decreasing the velocity boundary layer thickness. As the viscosity variation parameter is increased, the surface temperature tends to decrease, thus increasing the local Nusselt number. Moreover, the local Nusselt number of the elliptical cylinder increases as the Prandtl number of the fluid is increased.

1. Introduction

Natural convection heat transfer in boundary layer flow of viscous incompressible fluid from a horizontal cylinder represents an important problem because of their extensive engineering applications. Studies on natural convection boundary layer flow past a horizontal cylinder have been conducted by several researchers. Saville and Churchill [1] examined the laminar natural convection boundary layer flow near horizontal cylinders and vertical axisymmetric bodies. Merkin [2] studied the natural convection boundary layer flow over a cylinder of elliptic cross section. Lien et al. [3] examined the free convection heat transfer of micropolar fluid near a horizontal permeable cylinder at a non-uniform thermal condition. Bhattacharyya and Pop [4] studied the free convection heat transfer from an elliptical cylinder in micropolar fluids. Hossain et al. [5] examined the effect of thermal radiation on natural convection over cylinders of elliptic cross section. Mansour et al. [6] studied the coupled heat and mass transfer in magnetohydrodynamic flow of micropolar fluid on circular cylinders with uniform heat and mass flux. Cheng [7] studied the natural convection heat and mass transfer from a horizontal cylinder of elliptic cross section in micropolar fluid. Molla et al. [8] studied the natural convection flow from a horizontal cylinder with uniform heat flux in presence of heat generation.

The fluid viscosity may have a significant change with temperature, and it is thus necessary to consider the effects of the variation of viscosity on the heat transfer and fluid flow. Gray et al. [9] studied the effect of significant viscosity variation on heat transfer in water-saturated porous media. Lings and Dybbs [10] examined the forced convection over a flat plate in porous media with variable viscosity proportional to an inverse linear function of temperature. Kafoussius and Williams [11]

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studied the effect of temperature- dependent viscosity on the free convection laminar boundary layer flow along a vertical isothermal plate. Kafoussius and Rees [12] examined the effect of temperature-dependent viscosity on the mixed convection laminar boundary layer flow along a vertical isothermal plate. Molla et al. [13] studied the natural convection flow from an isothermal circular cylinder with temperature-dependent viscosity. Cheng [14] studied the double-diffusive convection from a vertical truncated cone in a porous medium with variable viscosity. Pal and Mondal [15] examined the influence of temperature-dependent viscosity and thermal radiation on MHD-forced convection over a non-isothermal wedge.

Motivated by the works above, this work uses a suitable coordinate transformation and the cubic spline collocation method to analyze the natural convection heat transfer from a horizontal cylinder of elliptic cross section with constant surface heat flux in a Newtonian fluid with temperature-dependent viscosity. The results obtained herein are compared with the solutions obtained by Merkin [2] for natural convection heat transfer from an elliptical cylinder in Newtonian fluids with constant surface heat flux to assess the accuracy of the solutions. This work studies the influence of the viscosity variation parameter, the aspect ratio, the Prandtl number and the orientation on the heat transfer characteristics near a horizontal cylinder of elliptic cross section with constant surface heat flux in Newtonian fluids with temperature-dependent viscosity. The solutions given in this work include the results in both cases when the major axis is horizontal (blunt orientation) and vertical (slender orientation).

2. Analysis

Consider the steady-state two-dimensional, laminar natural convection boundary layer flow of Newtonian fluid with temperature-dependent viscosity near a horizontal elliptical cylinder constant surface heat flux, as shown in Fig. 1, where a is the length of semi-major axis, b is the length of semi-minor axis for the elliptical cylinder. In this figure, A denotes the angle made by the outward normal from the cylinder with the downward vertical and Ω is the eccentric angle. Note that this work investigates horizontal elliptical cylinders of two different orientations. The orientation is blunt when the major axis is horizontal, as shown in Fig. 1, and the orientation is slender when the major axis is vertical. The surface of the elliptical cylinder is held at a constant surface heat flux q_w while the ambient fluid temperature is T_∞.

Based on the boundary layer and Boussinesq approximations, the governing for laminar boundary layer flow by natural convection of a Newtonian fluid with temperature-dependent viscosity near a horizontal cylinder of elliptical cross section subject to a constant surface heat flux can be written in two-dimensional Cartesian coordinates (x,y) as [2]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\rho g\beta(T-T_\infty)\sin A, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

The appropriate boundary conditions for the problem are:

$$u=v=0,\quad -k\frac{\partial T}{\partial y}=q_w \text{ on } y=0, \tag{4}$$

$$u=0,\quad T=T_\infty \text{ as } y\to\infty. \tag{5}$$

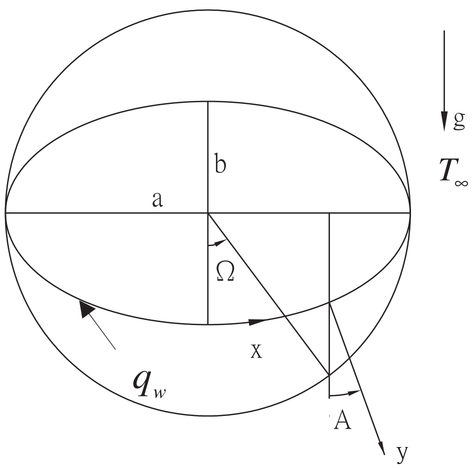

Fig. 1. Physical model and coordinate system for an elliptical cylinder of blunt orientation.