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Exact solutions of nonlinear evolution equations with variable coefficients using exp-function method

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ABSTRACT

In this paper, we apply the exp-function method to construct generalized solitary and periodic solutions of nonlinear evolution equations with variable coefficients. The proposed technique is tested on the Zakharov–Kuznetsov and (2+1)-dimensional Broer–Kaup equations with variable coefficients. These equations play a very important role in mathematical physics and engineering sciences. The suggested algorithm is quite efficient and is practically well suited for use in these problems. Obtained results clearly indicate the reliability and efficiency of the proposed exp-function method.

1. Introduction

It is well known that nonlinear complex physical phenomena are related to nonlinear partial differential equations (NLPDEs), which are involved in many fields from physics to biology, chemistry, mechanics, plasma physics, optical fibers, chemical kinematics, chemical physics and geochemistry, etc. As mathematical models of the phenomena, the investigation of exact solutions of NLPDEs will help one to understand these phenomena better. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations [1]. In recent years, new exact solutions may help to find new phenomena. A variety of powerful methods, such as the tanh–sech method [2,3], extended tanh method [4,5], sine–cosine method [6,7], Hirota method [8,9], homogeneous balance method [10,11], Jacobi elliptic function method [12,13], F-expansion method [14,15], homotopy perturbation method [16,17], variational iteration method [18,19] and non-perturbative method [20] were used to develop nonlinear dispersive and dissipative problems.

In the last decades great progress was made in the development of methods for obtaining exact solutions of nonlinear evolution equations but the progress achieved is not adequate. Because, from our point of view, there is no single best method to obtain exact solutions of nonlinear evolution equations of both type and each method has its merits and deficiencies depending on the researcher's practice and the understanding of the method utilized. Moreover, it can be said that all these methods are problem dependant, namely some methods work well with certain problems but not others. Therefore, it is rather significant to apply some well-known methods in the literature to nonlinear partial differential equations which are not solved with that method to search probably new exact solutions or to verify the existing solutions with different methods.

The goal of the present paper is to extend the exp-function method introduced by [21] to finding new solitary solutions, compact-like solutions and periodic solutions for nonlinear evolution equations in mathematical physics. Our first interest in the present work is in implementing the exp-function method to stress its power in handling nonlinear equations, so that one can apply it to models of various types of nonlinearity. The next interest is in the determination of exact travelling wave solutions for the nonlinear evolution equations with variable coefficients.

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The paper is organized as follows. In Section 2, the key idea of the exp-function method is described. In Sections 3 and 4, the proposed method is applied to solve the Zakharov–Kuznetsov and (2+1)-dimensional Broer–Kaup equations with variable coefficients. We conclude the paper in the last section.

2. Brief exp-function method

The exp-function method was first proposed by He and Wu in 2006 [21] and systematically studied in [22,23] and was successfully applied to a KdV equation with variable coefficients [24], to a class of nonlinear partial differential equations [25,26], to Burgers and combine KdV–mKdV (extended KdV) equations [27], to Schwarzian Korteweg–de Vries equation [28], to general improved KdV equation [29] to difference–differential equations [30,31] etc. We consider a general nonlinear PDE in the form

$$P(u, u_t, u_x, u_y, u_{xx}, u_{tt}, u_{yy}, \ldots) = 0. \tag{2.1}$$

Using a transformation

$$\xi = kx + ly + wt, \tag{2.2}$$

where k, l and w are constants, we can rewrite Eq. (2.1) in the following nonlinear ODE:

$$Q(u, u', u'', u''', \ldots) = 0, \tag{2.3}$$

where the prime denotes the derivation with respect to ξ.

According to exp-function method, we assume that the solution can be expressed in the form [21]

$$u(\xi) = \frac{\sum_{n=-c}^{d} a_n \exp(n\xi)}{\sum_{m=-p}^{q} b_m \exp(m\xi)}, \tag{2.4}$$

where c, d, p and q are positive integers which could be freely chosen, a_n and b_m are unknown constants to be determined. To determine the values of c and p, we balance the linear term of highest order in Eq. (2.3) with the highest order nonlinear term. Similarly to determine the values of d and q, we balance the linear term of lowest order in Eq. (2.3) with the lowest order nonlinear term.

We suppose that the solution of Eq. (2.3) can be expressed as

$$u(\xi) = \sum_{i=1}^{n} a_i \phi^i, \tag{2.5}$$

where ϕ is the solution of the sub-equation $\phi' = r + s\phi + t\phi^2$. In a similar way, ϕ can be expressed in the form:

$$\phi(\xi) = \frac{\sum\limits_{n=-c}^{d} a_n \exp(n\xi)}{\sum\limits_{m=-p}^{q} b_m \exp(m\xi)}. \tag{2.6}$$

To show the efficiency of the method described in the previous part, we present some examples.

3. The Zakharov–Kuznetsov equation with variable coefficients

We consider the Zakharov–Kuznetsov equation with variable coefficients [32]

$$u_t + \alpha(t)uu_x + \beta(t)u^2u_x + u_{xxx} + \gamma(t)u_{xyy} = 0. \tag{3.1}$$

Biswas and Zerrad are obtained an exact 1-soliton solution of the Zakharov–Kuznetsov equation, with power law nonlinearity, by the solitary wave ansatz method. A couple of conserved quantities of this equation are also calculated by using this 1-soliton solution in [33,34].

Introducing the transformation ξ defined as $\xi = kx + ly + \int \tau(t)dt$, we have

$$\tau(t)u' + k\alpha(t)uu' + k\beta(t)u^2u' + (k^3 + kl^2\gamma(t))u''' = 0. \tag{3.2}$$

Using the ansatz (2.4), for the linear term of highest order u''' with the highest order nonlinear term u^2u'. By simple calculation, we have

$$u''' = \frac{c_1 \exp[(-7p - c)\xi] + \cdots}{c_2 \exp[-8p\xi] + \cdots} = \frac{c_1 \exp[-(7p + c)\xi] + \cdots}{c_2 \exp[-8p\xi] + \cdots}, \tag{3.3}$$