Applied Mathematics and Computation 216 (2010) 3002–3015

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Permanence and extinction analysis for a delayed periodic predator–prey system with Holling type II response function and diffusion

Zijian Liu [a,*], Shouming Zhong [a,b]

[a] School of Applied Mathematics, University of Electronic Science and Technology of China, Chengdu 610054, PR China
[b] Key Laboratory for NeuroInformation of Ministry of Education, University of Electronic Science and Technology of China, Chengdu 610054, PR China

ARTICLE INFO

Keywords:
Lotka–Volterra system
Dispersal
Delay
Permanence
Periodic solution

ABSTRACT

In this paper, it is studied that two species predator–prey Lotka–Volterra type dispersal system with delay and Holling type II response function, in which the prey species can disperse among n patches, while the density-independent predator species is confined to one of the patches and cannot disperse. Sufficient conditions of integrable form for the boundedness, permanence, extinction and the existence of positive periodic solution are established, respectively.

1. Introduction

The effect of environment change in the growth and diffusion of species in a heterogeneous habitat is a subject of considerable interest in the ecological literature. Since the pioneering theoretical work by Skellam [20], many works have focused on the effect of spatial factors which play a crucial rule in the persistence and stability of a population [4,5,9,16,27,29,32]. And two-species predator–prey discrete dispersal systems have been extensively studied in many articles (see [8,11,14,23]). Most of the existing models deal with autonomous population systems and indicate that a dispersal process in an ecological system is often considered to have a stabilizing influence on the system [6], but it is also probably destabilizing the system [18]. In [16], Kuang and Takeuchi showed that discrete diffusions are capable of both stabilizing and destabilizing a given ecosystem. For time-dependent predator–prey systems in patchy environments, existing results have largely been restricted to permanence and extinction analysis due to the increased complexity of global analysis.

Owing to many natural and man-made factors such as low birth rate, high death rate, hunting, decreasing habitats, aggravating living environment, etc., some predator animals become very rare and even liable to extinction. Hence, in [8], Cui and Chen studied a time-dependent predator–prey system where the predator and prey disperse among patches, and showed that dispersal can make the prey and predator permanent even if the prey live in some poor patches. And in many other articles ([7,21,24]), authors usually assumed that the predator's density is regulated only by predation and is density-independent, which is much identical with the real biological background. So, in this paper, we will consider the instance that the predator's density is regulated only by predation and is density-independent.

On the other hand, based on experiments, in [12], Holling suggested three different kinds of functional responses for different kinds of species to model the phenomena of predation, which made the standard Lotka–Volterra system more realistic. For example, he proposed the form

$$\Phi(x) = \frac{mx}{a+x},$$

* Corresponding author.
E-mail addresses: hbliuzijian@126.com (Z. Liu), zhongsm@uestc.edu.cn (S. Zhong).

0096-3003/$ - see front matter
doi:10.1016/j.amc.2010.04.012

as a Holling type II response function, the study of this can be seen in many articles [1,19,28,30]. He also proposed the Holling type III response function in the following form:

$$\Phi(x) = \frac{mx^2}{a + x^2},$$

which also have been discussed in many articles no matter autonomous and nonautonomous, such as [13,17]. However, the effects of a periodically varying environment and time delay play an important role in the permanence and extinction of population dynamic systems (e.g., [2,3,7,10,15]). Thus, the assumptions of periodicity of the parameters and time delay of species during the course of dispersion and conversion of nutrients into the reproduction are effective way to characterize and investigate dispersal population systems. Moreover, in [31], the authors discussed two species time-delayed periodic predator–prey Lotka–Volterra type systems with dispersal but without response function, the authors get some sufficient conditions on the boundedness, permanence and existence of positive periodic solution for the systems, which enlighten us to study this type nonautonomous periodic predator–prey systems with dispersal and Holling type II response function, and to see wether can we get some good conditions or not.

Motivated by the arguments above, in this paper, we consider the following two species periodic predator–prey Lotka–Volterra type system with Holling type II response function:

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= x_1(t)\left[a_1(t) - b_1(t)x_1(t) - c(t)\int_{t-\sigma}^{t} k_1(t-s)\frac{y(s)}{1+mx_1(s)}ds\right] + \sum_{j=1}^{n} d_{1j}(t)[x_j(t) - x_1(t)],\\
\frac{dx_i(t)}{dt} &= x_i(t)[a_i(t) - b_i(t)x_i(t)] + \sum_{j=1}^{n} d_{ij}(t)[x_j(t) - x_i(t)], \quad i = 2,3,\ldots,n,\\
\frac{dy(t)}{dt} &= y(t)\left[-e(t) + f(t)\int_{t-\pi}^{t} k_2(t-s)\frac{x_1(s)}{1+mx_1(s)}ds\right],
\end{aligned} \tag{1.1}$$

where $t \in R_{+0} = [0,\infty)$, $x_i(i \in I = \{1,2,\ldots,n\})$ denote the population density of the prey species in the ith patch, y presents the population density of the predator species, $a_i(t)$, $b_i(t)$, the intrinsic growth rate and density-dependent coefficient of the prey in the ith patch, respectively, $c(t)$ the capturing rate of the predator, $e(t)$ the death rate of the predator, $f(t)$ the rate of conversion of nutrients into the reproduction of the predator and $d_{ij}(t)(i,j \in I, i \neq j)$ the dispersal rate of the prey species from the ith patch to the jth patch, m is a nonnegative constant, the term $x_1/(1 + mx_1)$ denotes the functional response of the predator. And the function $k_i(s)(i = 1,2)$, defined on $[0,\sigma]$ and $[0,\pi]$ respectively, are both nonnegative and integrable. That is, $\int_0^\sigma k_1(s)ds = 1$ and $\int_0^\pi k_2(s)ds = 1$. Then we know that the term $\int_{t-\sigma}^t k_1(t-s)y(s)/(1+mx_1(s))ds$ represents the negative effect to the growing rate of the prey population at time t due to the intervention of predator during the time $t-\sigma$ to t, and the term $\int_{t-\pi}^t k_2(t-s)x_1(s)/(1+mx_1(s))ds$ represents the positive effect to the growing rate of the predator population at time t due to the predation during the time $t-\pi$ to t. In this paper, we always assume that the functions $a_i(t)$, $b_i(t)$, $d_{ij}(t)(i,j \in I, i \neq j)$, $c(t)$, $e(t)$ and $f(t)$ are continuous and periodic defined on R_{+0} with common period $\omega > 0$ and $d_{ii} = 0 (i \in I)$ for all $t \in R_{+0}$.

Our main purpose is to establish a series of criteria on the ultimate boundedness, permanence, extinction and the existence of the periodic solution of the prey and predator species for system (1.1). The method used in this paper is motivated by the works given by Teng and Chen in [26], and Zhang and Teng in [31].

The organization of this paper is as follows. Section 2 presents some basic assumptions and useful lemmas. In Section 3, we state and prove the main results for two species periodic predator–prey Lotka–Volterra type system with Holling type II response function and diffusion. Finally, a conclusion is given in Section 4.

2. Preliminaries

In this section, we will give some preliminary knowledge that will be used in the following sections.

Let set $C_+ = \{\phi = (\phi_1,\phi_2,\ldots,\phi_{n+1}) \in C: \phi_i(i = 1,2,\ldots,n+1)$ is nonnegative on $[-\tau,0]$ and $\phi_i(0) > 0\}$, where $\tau = \max\{\pi,\sigma\}$. For ecological reasons, we always assume that solutions of system (1.1) satisfy the following initial conditions:

$$x_i(s) = \phi_i(s), \quad y(s) = \phi_{n+1}(s) \quad \text{for all } s \in [-\tau,0],\ i \in I, \tag{2.1}$$

where $\phi = (\phi_1,\phi_2,\ldots,\phi_{n+1}) \in C_+$. It is easy to prove that the functional of the right side system (1.1) is continuous and satisfy a local Lipschitz condition with respect to ϕ in the space $R \times C$. Therefore, by the fundamental theory of functional differential equations, for any $\phi \in C_+$ system (1.1) has a unique solution $(x(t,\phi),y(t,\phi)) = (x_1(t,\phi),x_2(t,\phi),\ldots,x_n(t,\phi),y(t,\phi))$ satisfying the initial condition (2.1). It is also easy to prove that the solution $(x(t,\phi),y(t,\phi))$ is positive, that is $x_i(t,\phi) > 0 (i \in I)$ and $y(t,\phi) > 0$ in its maximal interval of the existence. In this paper, such a solution of system (1.1) is called a positive solution.

We define that system (1.1) to be permanent, if there are constants $M \geqslant m > 0$ such that

$$m \leqslant \liminf_{t\to\infty} x_i(t) \leqslant \limsup_{t\to\infty} x_i(t) \leqslant M, \quad i \in I$$

and