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On the number of limit cycles of a Z_4-equivariant quintic polynomial system [☆]

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ABSTRACT

In this paper we study the number of limit cycles of a near-Hamiltonian system under Z_4-equivariant quintic perturbations. Using the methods of Hopf and heteroclinic bifurcation theory, we found that the perturbed system can have 13 limit cycles.

1. Introduction

The 16th Hilbert's problem consists of two parts. The second part of it concerns the number of limit cycles and their relative locations for polynomial vector fields. There have been many studies on this aspect, see [1–5]. Usually it is very hard to find the number of limit cycles for general polynomial systems. To reduce the difficulty one can study the systems with some symmetry. An important symmetry is the Z_q-equivariance which was first introduced in [6]. The following theorem was proved in [6].

Theorem 1.1. *The real polynomial system*

$$\frac{dx}{dt} = P_n(x,y), \quad \frac{dy}{dt} = Q_n(x,y), \tag{1.1}$$

where $P_n(x,y)$ and $Q_n(x,y)$ are polynomials of x and y with degree n, is Z_q-equivariant if the function $F(z,\bar{z}) = P_n(x,y) + iQ_n(x,y)$, where $x = \frac{1}{2}(z+\bar{z}),\ y = \frac{1}{2i}(z-\bar{z})$ and $i = \sqrt{-1}$, has the form

$$F(z,\bar{z}) = \sum_{l\geqslant 1} {}_l(|z|^2)\bar{z}^{lq-1} + \sum_{l\geqslant 0} h_l(|z|^2)\bar{z}^{lq+1} \equiv F_{n,q}(z,\bar{z}), \tag{1.2}$$

where p_l and h_l are complex functions. In addition, Eq. (1.1) is a Hamiltonian system having Z_q -equivariance if and only if Eq. (1.2) holds and $\frac{\partial F}{\partial z} + \frac{\partial \bar{F}}{\partial \bar{z}} \equiv 0$.

Yu and Han [7] proved that a cubic Z_2-equivariant system can have 12 limit cycles and that a cubic Z_5-equivariant system can have 1 limit cycle. Yu et al. [8] proved that a cubic Z_3-equivariant system can have three small limit cycles and one big limit cycle. Wu et al. [9] studied a Z_4-equivariant quintic system having at least 16 limit cycles. Zhang et al. [10] found a quartic system having at least 15 limit cycles. Li et al. [11] studied a quintic system, obtaining at least 24 limit cycles. More results can be found in [12–20].

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Now taking $n = 3$ and $q = 4$ in (1.2) we have

$$F_{3,4}(z,\bar{z}) = \left(A_0 + A_1|z|^2\right)z + A_3\bar{z}^3, \tag{1.3}$$

where A_0,A_1 and A_3 are complex numbers.

Let $F_{3,4}$ satisfy

$$\frac{\partial F_{3,4}}{\partial z} + \frac{\partial \overline{F}_{3,4}}{\partial \bar{z}} \equiv 0, \tag{1.4}$$

which implies ReA_0 = ReA_1 = 0. Then under (1.3) and (1.4) we see that the corresponding real cubic system has the form

$$\begin{cases} \dot{x} = -y\left[B_0 + (B_1 - 1)y^2 + (B_1 + 3)x^2\right] = H_y, \\ \dot{y} = x\left[B_0 + (B_1 - 1)x^2 + (B_1 + 3)y^2\right] = -H_x, \end{cases} \tag{1.5}$$

where

$$H(x,y) = -B_0(x^2 + y^2)/2 - (B_1 - 1)(x^4 + y^4)/4 - (B_1 + 3)x^2y^2/2.$$

For phase portraits of (1.5), see Fig. 1.

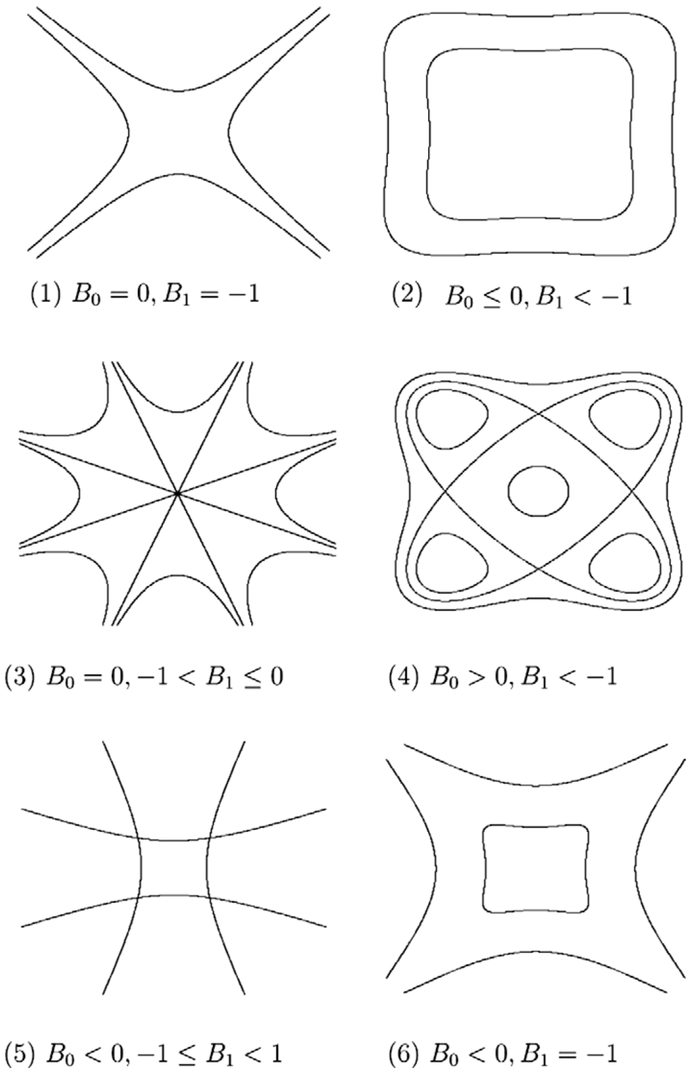

Fig. 1. The phase portrait of system (1.5).