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# Controlling hyperchaos in hyperchaotic Lorenz system using feedback controllers

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ABSTRACT

In this paper, the control strategies for hyperchaotic Lorenz system is investigated. The ordinary, dislocated, enhancing and speed feedback controls are used to suppress hyperchaos to unstable equilibrium. The Routh–Hurwitz criterion is used to derive the conditions of stability of the controlled hyperchaotic systems. It is found that the coefficients of enhancing feedback control and dislocated feedback control may be smaller than those of ordinary feedback control, so, the complexity and cost of the system control are reduced. Numerical simulations are given to illustrate the effectiveness of the proposed controllers.


## 1. Introduction

Dynamic chaos is a very interesting nonlinear effect, which has been intensively studied since Lorenz [1] found the first canonical chaotic attractor in 1963. The effect is very common, it has been detected in a large number of dynamic systems of various physical nature. However, this effect is sometimes undesirable in practice, and it restricts the operating range of many electronic and mechanic devices. In this case, therefore, it is necessary that the chaotic behavior should be controlled. Recently, controlling this kind of complex dynamical systems has attracted a great deal of attention within the engineering society. Research efforts have investigated the chaos control in many physical chaotic systems [2–13]. Recently, following the pioneer work of Ott et al. [2], different control strategies for stabilizing chaos have been proposed, such as Adaptive variable structure control [14], feedback control [15], and LMI approach [16]. Generally speaking, there are two main approaches for controlling chaos: feedback control and non-feedback control. Among them, the feedback control is especially attractive and has been commonly applied to practical implementation due to its simplicity in configuration and implementation.

However, most of the works mentioned so far involved mainly with low-dimensional chaos systems with only one positive Lyapunov exponent. Hyperchaotic systems, possessing more than one positive Lyapunov exponents, have more complex behaviors and abundant dynamics than chaotic system. How to realize control and synchronization of hyperchaotic systems is an interesting and challenging work. In Ref. [17], Yan suppressed a new hyperchaotic Chen system by using feedback control method. In Ref. [18], Dou et al. suppressed another new hyperchaotic system to unstable equilibrium.

Very recently, Wang et al. [19] generated a new hyperchaotic Lorenz system, which is described by the following mathematical model:

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = cx - y - xz + w, \\ \dot{z} = xy - bz, \\ \dot{w} = -dx, \end{cases} \tag{1}$$

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in which $a$, $b$, $c$ and $d$ are constant parameters. With the parameters $a = 10$, $b = 8/3$, $c = 28$, and $d = 5$ adopted, system (1) has two positive Lyapunov exponents, i.e. $\lambda_1 = 0.3997$ and $\lambda_2 = 0.3113$. And it has a unique unstable equilibrium $O(0,0,0,0)$. Thus, system (1) shows a hyperchaotic behaviour. In Ref. [20], based on the Lyapunov stability theory and adaptive control approach, Cai et al. studied adaptive control and synchronization problems of the hyperchaotic Lorenz system (1).

In this paper, we will control hyperchaos in the new hyperchaotic system (1). The ordinary feedback control, dislocated feedback control, enhancing feedback control, and speed feedback control are used to suppress hyperchaos to unstable equilibrium. Moreover, numerical simulations are applied to verify the effectiveness of chosen controllers.

## 2. Controlling hyperchaotic attractor to equilibrium *O*(0,0,0,0)

To control the hyperchaotic system (1) to the unstable equilibrium $O(0,0,0,0)$, we use the feedback control approach to control it. Let us assume that the controlled hyperchaotic system is given by

$$\begin{cases} \dot{x} = a(y - x) + u_1, \\ \dot{y} = cx - y - xz + w + u_2, \\ \dot{z} = xy - bz + u_3, \\ \dot{w} = -dx + u_4, \end{cases} \tag{2}$$

where $u_1$, $u_2$, $u_3$ and $u_4$ are external negative feedback control inputs which will be suitably derive the trajectory of the hyperchaotic system, specified by $(x, y, z, w)$ to the equilibrium $O(0,0,0,0)$ of uncontrolled system (i.e. $u_i = 0$, $i = 1,2,3,4$).

### *2.1. Ordinary feedback control*

For the ordinary feedback control, the system variable is often multiplied by a coefficient as the feedback gain, and the feedback gain is added to the right-hand of the corresponding equation.

**Theorem 1.** *Let $u_2 = -ky$, $u_1 = u_3 = u_4 = 0$, and the controlled hyperchaotic system be*

$$\begin{cases} \dot{x} = 10(y - x), \\ \dot{y} = 28x - y - xz + w - ky, \\ \dot{z} = xy - 8/3z, \\ \dot{w} = -5x, \end{cases} \tag{3}$$

*where $k$ is the feedback coefficient. When $k > 27.1311$, system (3) will gradually converge to the unstable equilibrium point $O(0,0,0,0)$.*

**Proof.** The Jacobi matrix of system (3) is

$$\mathbf{J} = \begin{pmatrix} -10 & 10 & 0 & 0 \\ 28 & -1-k & 0 & 1 \\ 0 & 0 & -8/3 & 0 \\ -5 & 0 & 0 & 0 \end{pmatrix}. \tag{4}$$

The characteristic equation of matrix (4) is

$$(\lambda + 8/3)(\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3) = 0,$$

where

$$c_1 = k + 11, \quad c_2 = 10k - 270, \quad c_3 = 50.$$

If

$$k > 27.1311, \tag{5}$$

then $c_1 > 0$, $c_3 > 0$ and $c_1c_2 - c_3 > 0$. According to the Routh–Hurwitz criterion, the Jacobian matrix (4) has four negative real part eigenvalues. Thus system (3) will gradually converge to the unstable equilibrium point $O(0,0,0,0)$. □

Numerical simulations are used to investigate the controlled hyperchaotic system (3) using fourth-order Runge–Kutta scheme. The feedback coefficient is given by $k = 30$. The initial values are taken as $[x(0) = -10,\ y(0) = 10,\ z(0) = 20,\ w(0) = 30]$. The behaviors of the states $(x(t), y(t), z(t), w(t))$ of the controlled hyperchaotic system (3) with time $t$ are displayed in Fig. 1. It is shown that the convergence rate of variable $w$ is the slowest.

Similarly, if the controller is chosen as $u_1 = -kx$ and $u_2 = u_3 = u_4 = 0$, when $k > 270.1778$, then system (3) will gradually converge to the unstable equilibrium point $O(0,0,0,0)$. If the single ordinary feedback controller is chosen as $u_3 = -kz$ and