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An iterative method for equilibrium problems and a finite family of relatively nonexpansive mappings in a Banach space

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ABSTRACT

In this paper, we introduce a new iterative method for finding a common element of the set of fixed points of a finite family of relatively nonexpansive mappings and the set of solutions of an equilibrium problem in uniformly convex and uniformly smooth Banach spaces. Then we prove a strong convergence theorem by using the generalized projection.

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1. Introduction

Let E be a real Banach space and let E^* be the dual space of E. Let C be a nonempty, closed and convex subset of E. Let f be a bifunction from $C \times C$ to the set of real numbers $\mathbb{R}$. The equilibrium problem is to find $\hat{x} \in C$ such that

$$f(\hat{x}, y) \geqslant 0, \quad \forall y \in C. \tag{1.1}$$

The set of solutions of (1.1) is denoted by $EP(f)$. Let T be a mapping from C into itself. We denote by $F(T)$ the set of fixed points of T.

Numerous problems in physics, optimization and economics can be reduced to find a solution of the equilibrium problem (1.1) under the constraint on the common fixed points set of a family of mappings $\{T_i\}_{i=1}^m$. Such problems are equivalent to the problems of finding a common element of the solutions set of the equilibrium problem and the fixed points set of a family of mappings.

In this paper, we deal with a class of relatively nonexpansive mappings introduced by Matsushita and Takahashi [10]. There are a lot of important examples of relatively nonexpansive mappings such as the generalized projections, resolvents of a maximal monotone operators, and others; see [9,10].

In 2005, Matsushita and Takahashi [9] introduced an iterative process to approximate a fixed point of a single relatively nonexpansive mapping. They proved strong convergence theorems by using the generalized projection in a uniformly convex and uniformly smooth Banach space. Two years later, Plubtieng and Ungchittrakool in [14] generalized the previous iterative method with two relatively nonexpansive mappings.

In 2007, Tada and Takahashi [18] introduced strong convergence theorems for finding common elements of the set of fixed points of a nonexpansive mapping and the set of solutions of an equilibrium problem in a Hilbert space. Later, Takahashi and Zembayashi in [22] modified the iterative method to find a common element of the set of fixed points of a relatively nonexpansive mapping and the set of solutions of an equilibrium problem in a Banach space. Recently, the generalized equilibrium problem was also considered by Takahashi and Takahashi [21].

Motivated and inspired by the previous authors, we introduce a new iterative scheme and prove a strong convergence theorem of such a scheme for finding a common element of the set of solutions of an equilibrium problem and the set of fixed points of a finite family of relatively nonexpansive mappings in a Banach space. Our main result extends the results in [9,11,14,18,22] in several aspects.

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2. Preliminaries

Let E be a real Banach space with norm $\|\cdot\|$ and let E^* be the dual space of E. The normalized duality mapping $J : E \to 2^{E^*}$ is defined by

$$J(x) = \{ x^* \in E^* : \langle x, x^* \rangle = \|x\|^2 = \|x^*\|^2 \}$$

for all $x \in E$. A Banach space E is said to be *strictly convex* if $\|\frac{x+y}{2}\| < 1$ for all $x, y \in E$ with $\|x\| = \|y\| = 1$ and $x \neq y$. It is also said to be *uniformly convex* if $\lim_{n\to\infty}\|x_n - y_n\| = 0$ for any two sequences $\{x_n\}$, $\{y_n\}$ in E such that $\|x_n\| = \|y_n\| = 1$ and $\lim_{n\to\infty}\|\frac{x_n+y_n}{2}\| = 1$. Let $S = \{x \in E : \|x\| = 1\}$ be the unit sphere of E. A Banach space E is said to be *smooth* provided that $\lim_{t\to 0}\frac{\|x+ty\|-\|x\|}{t}$ exists for each $x, y \in S$. It is also said to be *uniformly smooth* if the limit is attained uniformly for $x, y \in S$. Moreover, E is said to have a *uniformly Gâteaux differentiable norm* if for each $y \in S$, the limit is attained uniformly for $x \in S$. The norm of E is said to be *Fréchet differentiable*, if for each $x \in S$, the limit is attained uniformly for $y \in S$. It is well-known that if E is smooth, then the duality mapping J is single valued and if E is uniformly smooth, then J is uniformly norm-to-norm continuous on each bounded subsets of E, i.e., for a bounded set C of E and $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\|x - y\| < \delta \Rightarrow \|Jx - Jy\| < \varepsilon$$

for all $x, y \in C$; see [5,15–17,19] for more details. It is also known that, if E is uniformly convex, then E has the *Kadec–Klee property*, that is, $x_n \rightharpoonup x$ and $\|x_n\| \to \|x\|$ imply $x_n \to x$ for $x \in E$ where $x_n \rightharpoonup x$ and $x_n \to x$ denote the weak convergence and strong convergence of the sequence $\{x_n\}$ to x, respectively. Let E be a smooth Banach space. The function $\phi : E \times E \to \mathbb{R}$ is defined by

$$\phi(x, y) = \|x\|^2 - 2\langle x, Jy \rangle + \|y\|^2$$

for all $x, y \in E$; see [19,20] for more details. Then, it is obvious by the definition that $\phi(x,y) \leqslant \|x\|\|Jx - Jy\| + \|y - x\|\|y\|$. Moreover, $\phi(x,y) = \|x - y\|^2$ for all x, y in a Hilbert space H. Let C be a closed and convex subset of E and let T be a mapping from C into itself. A mapping T is called *nonexpansive* if $\|Tx - Ty\| \leqslant \|x - y\|$ for all $x, y \in C$ and *relatively nonexpansive* if T satisfies the following conditions:

(1) $F(T)$ is nonempty;
(2) $\phi(u, Tx) \leqslant \phi(u, x)$, for all $u \in F(T)$ and $x \in C$;
(3) $\widehat{F}(T) = F(T)$ where $\widehat{F}(T)$ is the set of all asymptotic fixed points of T; see [9,10,22] for more details.

The following lemmas are so useful in proving our main theorem.

Lemma 2.1 (*Kamimura and Takahashi [7]*). *Let E be a uniformly convex and smooth Banach space and let $\{x_n\}$ and $\{y_n\}$ be two sequences in E such that either $\{x_n\}$ or $\{y_n\}$ is bounded. If $\lim_{n\to\infty}\phi(x_n, y_n) = 0$, then $\lim_{n\to\infty}\|x_n - y_n\| = 0$.*

Let C be a nonempty, closed and convex subset of E. Suppose that E is reflexive, strictly convex and smooth. Then, for any $x \in E$, there exists a point $x_0 \in C$ such that $\phi(x_0, x) = \min_{y\in C}\phi(y, x)$. The mapping $\Pi_C : E \to C$ defined by $\Pi_C x = x_0$ is called the *generalized projection* [1,2,7]. The following are well-known results. For example, see [1,2,7].

Lemma 2.2. [Alber [1], Alber and Reich [2], Kamimura and Takahashi [7]]*Let C be a nonempty closed convex subset of a smooth, strictly convex and reflexive Banach space E, let $x \in E$ and let $x_0 \in C$. Then $x_0 = \Pi_C x$ if and only if*

$$\langle y - x_0, Jx - Jx_0 \rangle \leqslant 0, \quad \forall y \in C.$$

Lemma 2.3. [Alber [1], Alber and Reich [2], Kamimura and Takahashi [7]]*Let C be a nonempty closed convex subset of a smooth, strictly convex and reflexive Banach space E. Then*

$$\phi(x, \Pi_C y) + \phi(\Pi_C y, y) \leqslant \phi(x, y) \quad \forall x \in C \text{ and } y \in E.$$

Lemma 2.4. [Kamimura and Takahashi [7]]*Let E be a smooth and uniformly convex Banach space and let $r > 0$. Then there exists a continuous, strictly increasing, and convex function $g : [0, 2r] \to \mathbb{R}$ such that $g(0) = 0$ and*

$$g(\|x - y\|) \leqslant \phi(x, y)$$

for all $x, y \in B_r(0)$, where $B_r(0) = \{z \in E : \|z\| \leqslant r\}$.

Lemma 2.5. [Cho et al. [4]] *Let E be a uniformly convex Banach space and let $r > 0$. Then there exists a continuous, strictly increasing, and convex function $g : [0, \infty) \to [0, \infty)$ such that $g(0) = 0$ and*

$$\|\alpha x + \beta y + \gamma z\|^2 \leqslant \alpha\|x\|^2 + \beta\|y\|^2 + \gamma\|z\|^2 - \alpha\beta g(\|x - y\|)$$

for all $x, y, z \in B_r(0)$ and $\alpha, \beta, \gamma \in [0,1]$ with $\alpha + \beta + \gamma = 1$.

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