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Exp-function method for traveling wave solutions of nonlinear evolution equations

Muhammad Aslam Noor [a,b], Syed Tauseef Mohyud-Din [c,*], Asif Waheed [a], Eisa A. Al-Said [b]

[a] Department of Mathematics, COMSATS Institute of Information Technology, Islamabad, Pakistan
[b] Mathematics Department, College of Science, King Saud University, Riyadh, Saudi Arabia
[c] HITEC University, Taxila Cantt, Pakistan

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ABSTRACT

In this paper, we apply the exp-function method to construct generalized solitary and periodic solutions of nonlinear evolution equations. The proposed technique is tested on the modified Zakharov–Kuznetsov (ZK) and Zakharov–Kuznetsov-Modified-Equal-Width (ZK-MEW) equations. These equations play a very important role in mathematical physics and engineering sciences. The suggested algorithm is quite efficient and is practically well suited for use in these problems. Numerical results clearly indicate the reliability and efficiency of the proposed exp-function method.

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1. Introduction

This paper is devoted to the study of nonlinear evolution equations which arise in number of scientific models including the propagation of shallow water waves, long waves and chemical reaction diffusion models, fluid mechanics, astrophysics, solid state physics, plasma physics, chemical kinematics, chemical chemistry, optical fiber and geochemistry, see [2,3,6–9,11] and the references therein. A substantial amount of work has been invested for solving the governing equations of these physical models. Several techniques including method of characteristics, Riemann invariants, combination of waveform relaxation and multi grid, periodic multi grid wave form, inverse scattering, tanh, Sine-Cosine and homogeneous balance [6,11–15] have been used for the solutions of such problems. Most of these techniques encounter the inbuilt deficiencies and involve huge computational work. It is to be highlighted that Biswas and Zerrad [2,3] successfully and elegantly calculated soliton solutions of the nonlinear evolution equations. He and Wu [5] developed the exp-function method to seek the solitary, periodic and compacton like solutions of nonlinear differential equations. The method has been implemented on dispersive equations, KdVs, Boussinesq equations, master PDES, Kuramoto–Sivashinsky, Burger's equations, combined KdV and mKdV, Hybrid–Lattice system and discrete mKdV lattice, see [1–22] and the references therein. This clearly indicates that exp-function method is very effective and reliable. The basic motivation of this paper is to apply this reliable technique on nonlinear evolution equations. In particular, the proposed exp-function method is tested on the third-order nonlinear evolution equation which is called the modified Zakharov–Kuznetsov (ZK) equation and is of the form

$$u_t + u^2 u_x + u_{xxx} + u_{xyy} = 0 \tag{1}$$

and another third-order nonlinear equation which is called the Zakharov–Kuznetsov-Modified-Equal-Width (ZK-MEW) equation:

* Corresponding author.
E-mail addresses: noormaslam@hotmail.com (M.A. Noor), syedtauseefs@hotmail.com (S.T. Mohyud-Din), waheedasif@hotmail.com (A. Waheed), eisasaid@ksu.edu.sa (E.A. Al-Said).

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$$u_t + a\left(u^3\right)_x + u_{xx} + (bu_{xt} + ru_{yy})_x = 0, \tag{2}$$

where a, b and r are unknowns constants. The numerical results clearly indicate reliability and efficiency of the proposed exp-function method.

2. Exp-function method

We consider the general nonlinear PDE of the type

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, u_{xt}, u_{xy}, u_{ty} \ldots) = 0. \tag{3}$$

Using a transformation

$$\eta = kx + \omega y + \rho t, \text{ or } \eta = \alpha x + \beta y + \rho t, \tag{4}$$

where k, ω, α, β and ρ are constants, we can rewrite Eq. (3) in the following nonlinear ODE;

$$Q(u, u', u'', u''', \ldots) = 0. \tag{5}$$

According to Exp-function method, which was developed by He and Wu [5], we assume that the wave solution can be expressed in the following form

$$u(\eta) = \frac{\sum_{n=-d}^{c} a_n \exp[n\eta]}{\sum_{m=-q}^{p} b_m \exp[m\eta]}, \tag{6}$$

where p, q, c and d are positive integers which are known to be further determined, a_n and b_m are unknown constants. We can rewrite Eq. (6) in the following equivalent form.

$$u(\eta) = \frac{a_c \exp[c\eta] + \cdots + a_{-d} \exp[-d\eta]}{b_p \exp[p\eta] + \cdots + b_{-q} \exp[-q\eta]}. \tag{7}$$

This equivalent formulation plays an important and fundamental part for finding the analytic solution of problems. To determine the value of c and p, we balance the linear term of highest order of equation Eq. (5) with the highest order nonlinear term. Similarly, to determine the value of d and q, we balance the linear term of lowest order of Eq. (4) with lowest order nonlinear term [5–7,13].

3. Numerical applications

In this section, we apply the exp-function method developed by He and Wu [5] to construct generalized solitary and periodic solutions of the Zakharov–Kuznetsov (ZK) and Zakharov–Kuznetsov-Modified-Equal-Width (ZK-MEW) equations.

Example 3.1 [13]. Consider the modified Zakharov–Kuznetsov (ZK) equation (1)

$$u_t + u^2 u_x + u_{xxx} + u_{xyy} = 0.$$

Introducing a transformation as $\eta = \alpha x + \beta y + \rho t$, we can covert Eq. (1) into ordinary differential equations as

$$\rho u' + \alpha u^2 u' + (\alpha^3 + \alpha\beta^2) u''' = 0. \tag{8}$$

The solution of Eq. (8) can be expressed as follows

$$u(\eta) = \frac{a_c \exp[c\eta] + \cdots + a_{-d} \exp[-d\eta]}{b_p \exp[p\eta] + \cdots + b_{-q} \exp[-d\eta]}. \tag{7}$$

To determine the value of c and p, we balance the linear term of highest order of Eq. (7) with the highest order nonlinear term

$$u''' = \frac{c_1 \exp[(7p + c)\eta] + \cdots}{c_2 \exp[8p\eta] + \cdots}, \tag{9}$$

and

$$u^2 u' = \frac{c_3 \exp[(p + 3c)\eta] + \cdots}{c_4 \exp[4p\eta] + \cdots} = \frac{c_3 \exp[(5p + 3c)\eta] + \cdots}{c_4 \exp + [8p\eta] \cdots}, \tag{10}$$

where c_i are determined coefficients only for simplicity; balancing the highest order of exp-function in (9) and (10), we have

$$7p + c = 5p + 3c, \tag{11}$$

which in turn gives

$$p = c. \tag{12}$$

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