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Bifurcation studies on travelling wave solutions for an integrable nonlinear wave equation

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ABSTRACT

By using the method of planar dynamical systems to an integrable nonlinear wave equation, the existence of periodic travelling wave, solitary wave and kink wave solutions is proved in the different parametric conditions. The phase portraits of the travelling wave system are given. It can be shown that the existence of singular curves in the travelling wave system is the reason why the travelling wave solutions lose their smoothness. Moreover, the so-called W/M-shaped solitary wave solutions are obtained.

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1. Introduction

In this paper, we consider the following integrable nonlinear wave equation

$$(u + au_{xx})_t = bu_x + \frac{1}{2}[(u^2 + au_x^2)(u + au_{xx})]_x, \tag{1.1}$$

where $a = \pm 1,\ b$ is a constant. It is obtained through a reshuffling procedure of the Hamiltonian operators underlying the bi-Hamiltonian structure of KdV and mKdV equations [2,3]. In [3], Rosenau has studied the nonanalytic solitary waves of Eq. (1.1) and pointed out the interaction of nonlinear dispersion with nonlinear convection generates exactly compact structures. In this paper, we consider bifurcation problem of solitary waves, kink (or anti-kink) waves and periodic waves of Eq. (1.1) by using the bifurcation theory of planar dynamical systems.

We look for travelling wave solutions of Eq. (1.1) in the form of

$$u(x,t) = \phi(x - ct) = \phi(\xi), \tag{1.2}$$

where c is the wave speed. The travelling variable (1.2) permits us reducing Eq. (1.1) to an ODE for $\phi(\xi)$

$$-c(\phi + a\phi'')' = b\phi' + \frac{1}{2}[(\phi^2 + a{\phi'}^2)(\phi + a\phi'')]', \tag{1.3}$$

where ϕ' is the derivative with respect to ξ. Integrating Eq. (1.3) once, we have

$$-c(\phi + a\phi'') = b\phi + \frac{1}{2}(\phi^2 + a{\phi'}^2)(\phi + a\phi'') + d, \tag{1.4}$$

where d is an integral constant.

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Obviously, Eq. (1.4) is equivalent to the following two-dimensional system

$$\frac{d\phi}{d\xi}=y, \quad \frac{dy}{d\xi}=-\frac{2d+2(b+c)\phi+\phi^3+a\phi y^2}{a(\phi^2+ay^2+2c)}, \tag{1.5}$$

which has the first integral

$$H(\phi,y)=\frac{a}{2}\phi^2y^2+2d\phi+(b+c)\phi^2+\frac{1}{4}\phi^4+\frac{1}{4}y^4+cy^2=h. \tag{1.6}$$

System (1.5) is a planar dynamical system depending on its parameters. Because the phase orbits defined by the vector field of this system determine all its travelling wave solutions, we will investigate bifurcations of the phase portraits of the system (1.5), as some parameters are varied in phase plane (ϕ,y). Here, we should point out that we are considering physical model where only bounded travelling waves are meaningful. Hence, we are only concerned with bounded solutions of Eq. (1.1).

Suppose that $u(x,t)=\phi(x-ct)=\phi(\xi)$ is a continuous solution of Eq. (1.1) for $\xi\in(-\infty,\infty)$ and $\lim_{\xi\to+\infty}\phi(\xi)=a,\ \lim_{\xi\to-\infty}\phi(\xi)=b$. Usually, we have such results: (i) $\phi(x,t)$ is a solitary wave solution if $a=b$; (ii) $\phi(x,t)$ is a kink or anti-kink solution if $a\neq b$. According to above analysis, it can be concluded that a solitary wave solution of Eq. (1.1) corresponds to a homoclinic orbit of system (1.5), a kink (or anti-kink) wave solution Eq. (1.1) corresponds to a heteroclinic orbit (or the so-called connecting orbit) of system (1.5). Similarly, a periodic orbit of system (1.5) corresponds to a periodically travelling wave solution of Eq. (1.1). Thus, to investigate all possible bifurcations of solitary waves, kink wave and periodic waves of Eq. (1.1), we need to find all homoclinic orbits, heteroclinic orbit and periodic annuli of system (1.5), which depend on the system parameters. The bifurcation theory of dynamical systems [1,4,5] plays an important role in our study.

We noticed that right-hand side of the second equation of system (1.5) has a factor $(\phi^2+ay^2+2c)^{-1}$, so there exist singular curves $\phi^2+y^2+2c=0$ for $a=1,\ c<0$ and $\phi^2-y^2+2c=0$ for $a=-1$, respectively. On the two singular curves, $y_\xi=\phi_{\xi\xi}$ becomes unbounded or undefined. It implies that a smooth system (1.1) may has non-smooth travelling wave solutions. The occurrence of breaking for wave solutions of Eq. (1.1) (i.e. the phenomenon that a wave remains bounded, but its slope becomes unbounded in a finite time) has been studied before, in for instance [5,6], It has been pointed out that travelling waves sometimes lose their smoothness during the propagation due to the existence of such singular curves within the solution surfaces of the wave equation.

The paper is organized as follows. In Section 2, we discuss bifurcations of phase portraits of (1.5). In Section 3, we consider the existence of travelling wave solutions of Eq. (1.1). In Section 4, we study the M/W-shaped solitary wave solutions of Eq. (1.1). A short conclusion is given in Section 5.

2. Bifurcations of phase portraits of system (1.5)

In this section, we study all bifurcations of phase portrait in the parametric space. Denote $d\xi=a(\phi^2+ay^2+2c)d\zeta$, then system (1.5) has the same topological phase portraits as the following polynomial system

$$\frac{d\phi}{d\zeta}=a(\phi^2+ay^2+2c)y, \quad \frac{dy}{d\zeta}=-(2d+2(b+c)\phi+\phi^3+a\phi y^2), \tag{2.1}$$

except for the singular curves $\phi^2+ay^2+2c=0$. Easy to see that system (2.1) is a Hamiltonian system with Hamiltonian $H(\phi,y)$ defined as the same as (1.6). For a given $h=H(\phi,y)$, (1.6) determine a set of invariant curves of system (2.1), which contain some different branches of curves. As h is varied, (1.6) defined different families of orbits of system (2.1) with different dynamical behaviors.

Denote that

$$F(\phi)=\phi^3+2(b+c)\phi+2d. \tag{2.2}$$

It is easy to know that on the (ϕ,y)-phase plane, the abscissas of equilibrium points of system (2.1) on the ϕ-axis are the zeros of $F(\phi)$. Note that $F'(\phi)=3\phi^2+2(b+c), F'(\phi)$ has two zeros $\phi^*_\pm=\pm\sqrt{-\frac{2(b+c)}{3}}$ if $b+c<0$. Obviously,

$$F(\phi^*_\pm)=2d\pm\frac{4(b+c)}{3}\sqrt{-\frac{2(b+c)}{3}}. \tag{2.3}$$

According to (2.3), we can obtain the following bifurcation curves on the (b,d)-parameter plane for fixed c (see Fig. 1)

$$L:\ d=\pm\left[-\frac{2(b+c)}{3}\right]^{\frac{3}{2}}. \tag{2.4}$$

Throughout we assume that $d\geqslant 0$. Otherwise, we can make a transformation to reduce (1.5) to this case, so we consider only the upper half plane of (b,d)-parametric plane. The bifurcation curves L divides the (b,d)-parametric plane into two subregions:

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