Applied Mathematics and Computation 217 (2010) 1415–1420

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Finding general and explicit solutions (2 + 1) dimensional Broer–Kaup–Kupershmidt system nonlinear equation by exp-function method

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ARTICLE INFO

Keywords:
Exp-function method
(2 + 1) Dimensional Broer–Kaup–Kupershmidt equation
Nonlinear equation

ABSTRACT

In this work, we implement a relatively new analytical technique, the exp-function method, for solving nonlinear special form of generalized nonlinear (2 + 1) dimensional Broer–Kaup–Kupershmidt equation, which may contain high nonlinear terms. This method can be used as an alternative to obtain analytic and approximate solutions of different types of fractional differential equations which applied in engineering mathematics. Some numerical examples are presented to illustrate the efficiency and reliability of exp method. It is predicted that exp-function method can be found widely applicable in engineering.

1. Introduction

Nonlinear phenomena play important roles in applied mathematics, physics and also in engineering problems which each parameter varies depending on different factors. Solving nonlinear equations may guide authors to know the described process deeply and sometimes leads them to know some facts which are not simply understood through common observations. Moreover, obtaining exact solutions for these problems is a great purpose which has been quite untouched. However, in recent years, numerical analysis [1] has considerably been developed to be used for nonlinear partial equations such as (2 + 1) dimensional Broer–Kaup–Kupershmidt equation that have special kind of solutions. Using physical interpretation, solitary waves and property of a finite number of quantities existence conserved by the solutions.

The (2 + 1) dimensional Broer–Kaup–Kupershmidt system is in the form of [2]:

$$\begin{cases} u_{ty} - u_{xxy} + 2(uu_x)_y + 2v_{xx} = 0, \\ v_t + v_{xx} + 2(uv)_x = 0. \end{cases} \tag{1}$$

In addition, in recent years, scientists have presented some new methods for solving nonlinear partial differential equations; for instance, Bäcklund transformation method [3], Lie group method [4], Adomian's decomposition method [5], inverse scattering method [6], Hirota's bilinear method [7], homotopy analysis method [8,9] and He's HPM [10,11] and VIM [12–14]. In this Letter, we purpose to present implementation of exp-function method [15,16] to (2 + 1) dimensional Broer–Kaup–Kupershmidt equation. Having the available exact solution of the special form of the corresponding equations would provide us to have an admissible comparison of the results, which supports the applicability, accuracy and efficiency of the proposed methods.

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0096-3003/$ - see front matter
doi:10.1016/j.amc.2009.05.069

2. Basic idea of exp-function method

We first consider nonlinear equation in form:

$$N(u, u_t, u_x, u_{xx}, u_{yy}, u_{tx}, \ldots) = 0, \tag{2}$$

Introduction a complete variation defines as:

$$\eta = x + \beta y + \lambda t, \quad v = V(\eta), \quad u = U(\eta), \tag{3}$$

And therefore, the Eq. (1) constructs the ODE in form:

$$N(U, \lambda U', U', U'', \beta^2 U'', \lambda U'', \ldots) = 0. \tag{4}$$

And then solution of $U(\eta)$is form:

$$U(\eta) = \frac{\sum_{n=-c}^{d} a_n \exp(n\eta)}{\sum_{m=-p}^{q} b_m \exp(m\eta)} = \frac{a_c \exp(c\eta) + \cdots + a_{-d}\exp(-d\eta)}{a_p \exp(p\eta) + \cdots + a_{-q}\exp(-q\eta)}, \tag{5}$$

where c, d, p and q are positive integers which are unknown to be further determined, a_n and b_n are unknown constants.

3. Application to (2 + 1) dimensional Broer–Kaup–Kupershmidt system

Introducing a complex variation η defined as Eq. (3), and then Eq. (1) becomes an ordinary differential equation, which forms:

$$\beta\lambda U'' - \beta U''' + 2\beta(U'' + UU'') + 2V'' = 0, \tag{6}$$

$$\lambda V' - V'' + 2(UV' + U'V) = 0. \tag{7}$$

Integrating Eq. (6) twice leads to obtain $V(\eta)$

$$V = -\frac{\beta\lambda}{2}U + \frac{\beta}{2}U' - \frac{\beta}{2}U^2 + \frac{C}{2}\eta + \frac{C_1}{2}, \tag{8}$$

For simplicity, we set $C = 0$ then inserting Eq. (8) into Eq. (7) yields:

$$\left(C_1 + \frac{\beta\lambda^2}{2}\right)U' + \beta\lambda U'' - \frac{\beta}{2}UU''' + 2\beta\left(U'^2 + UU''\right) - 3\beta\lambda UU' - 3\beta U^2 U' = 0. \tag{9}$$

Integrating Eq. (9) once leads:

$$-\lambda\beta U^2 - \beta U^3 + C_1 U - \frac{\lambda^2\beta}{2}U + \lambda\beta U' + 2\beta UU' - \frac{\beta}{2}U'' = 0. \tag{10}$$

In order to determine values of c and p, we balance the linear term of the highest order UU''' with the highest order nonlinear term U^3 in Eq. (10), we have:

$$UU''' = \frac{c_1 \exp[(7p + c)\eta] + \ldots}{c_2 \exp[8p\eta] + \cdots}, \tag{11}$$

$$U^3 = \frac{c_3 \exp[(3c)\eta] + \ldots}{c_4 \exp[3p\eta] + \cdots} \times \frac{\exp[5p\eta]}{\exp[5p\eta]} = \frac{c_3 \exp[(3c + 5p)\eta] + \cdots}{c_4 \exp[8p\eta] + \cdots}, \tag{12}$$

where c_i are determined coefficients only for simplicity. Balancing highest order of exp-function, in Eq. (11) and Eq. (12), we have:

$$7p + c = 5p + 3c, \tag{13}$$

which leads to the result:

$$p = c. \tag{14}$$

Similarly to determine values of d and q, we balance the linear term of lowest order in Eq. (10)

$$UU''' = \frac{\cdots + d_1 \exp[-(7q + d)\eta]}{\cdots + d_2 \exp[-8q\eta]} \tag{15}$$

and

$$U^3 = \frac{\cdots + d_3 \exp[-(3d)\eta]}{\cdots + d_4 \exp[-3q\eta]} \times \frac{\exp[-5q\eta]}{\exp[-5q\eta]} = \frac{\cdots + d_3 \exp[-(3d + 5q)\eta]}{\cdots + d_4 \exp[-8q\eta]}, \tag{16}$$