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Generalized solitary and periodic wave solutions to a (2 + 1)-dimensional Zakharov–Kuznetsov equation

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ABSTRACT

In this paper, the Exp-function method is employed to the Zakharov–Kuznetsov equation as a (2 + 1)-dimensional model for nonlinear Rossby waves. The observation of solitary wave solutions and periodic wave solutions constructed from the exponential function solutions reveal that our approach is very effective and convenient. The obtained results may be useful for better understanding the properties of two-dimensional coherent structures such as atmospheric blocking events.

1. Introduction

Solving nonlinear evolution equations (NLEEs) has become a valuable task in many scientific areas including applied mathematics as well as the physical sciences and engineering in the last four decades or so. For this purpose, some accurate techniques have been presented in the open literature; for example, inverse scattering transform method [1], Jacobi elliptic function method [2], tanh–coth function method [3], sine–cosine function method [4], symmetry method [5], F-expansion method [6], Hirota's bilinear method [7], Painlevé expansion method [8], homogeneous balance method [9], Bäcklund transformation method [10], Adomian decomposition method [11], variational iteration method [12], homotopy analysis method [13], homotopy perturbation method [14] and so on. On the other hand, with the development of computer algebra systems (they allow us to perform the tedious and complicated algebraic calculations on a computer) in recent years, many direct and effective methods using symbolic computation are also presented such as the (G'/G)-expansion method [15–19] and the Exp-function method [20–24].

It is well known that many important dynamics processes can be described by specific nonlinear partial differential equations. In 1974, Zakharov and Kuznetsov [25] derived an equation which describes weakly nonlinear ion-acoustic waves in a strongly magnetized lossless plasma composed of cold ions and hot isothermal electrons. The Zakharov–Kuznetsov (ZK) equation is also known as one of two-dimensional generalizations of the KdV equation, another one being the Kadomtsev–Petviashvili (KP) equation for example. In contrast to the KP equation, the ZK equation is non-integrable by the inverse scattering transform method, though Shivamoggi [26] showed that it posses the Painlevé property by making a Painlevé analysis of the ZK equation. The ZK equation has also been derived in the context of plasma physics [27,28]. Biswas and Zerrad [29] considered the ZK equation with dual-power law nonlinearity and obtained 1-soliton solution by using the solitary wave ansatze.

The ZK equation is a very attractive model equation for the study of vortices in geophysical flows since it supports stable lump solitary waves [30]. Thus, more recently, to study the dynamics of two-dimensional coherent structures in planetary atmospheres and oceans, Gottwald [31] derived the ZK equation for large scale motion from the barotropic quasigeostrophic

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equation as a two-dimensional model for Rossby waves. The (2 + 1)-dimensional Zakharov–Kuznetsov ((2 + 1)D-ZK) equation [31] reads

$$u_t + \delta u_x + \alpha u u_x + \beta u_{xxx} + \gamma u_{xyy} = 0, \tag{1}$$

where δ, α, β, and γ are nonzero arbitrary constants and $u = u(x,y,t)$. He [32] applied the homotopy perturbation method to the (2 + 1)D-ZK equation (1) to search for traveling wave solutions. Using a sub-equation method (the elliptic equation is taken as a transformation), the traveling wave solutions for the (2 + 1)D-ZK equation (1) are also studied by Fu et al. [33]. There has not been much research on this special form (2 + 1)D-ZK equation (1) in the literature. The main goal of our present work is to further analyze this less studied form of the ZK equation by using the so-called Exp-function method.

Based on He and Wu's pioneer work [20] and his followers the Exp-function method has found some popularity in a research community, and there has been a number of papers refining the initial idea [34–39]. The Exp-function "method" consists of trying rational combinations of exponential functions as an "ansatz" to find exact solutions of the ODE for traveling waves of the original equation. The method is powerful for it can take full advantage of computer algebra systems, the solution procedure is actually almost impossible without using a computer.

2. The Exp-function method based on the symbolic computation

To begin with, suppose that we have a nonlinear partial differential equation for $u(x,y,t)$ in the form

$$P(u, u_t, u_x, u_y, u_{tt}, u_{tx}, u_{ty}, u_{xx}, u_{xy}, u_{yy}, \ldots) = 0, \tag{2}$$

where P is a polynomial in its arguments. We give an algorithmic description of our method as follows:

Step 1. (*Reduce NPDE to nonlinear ODE*) By taking $u(x,y,t) = U(\zeta)$, $\zeta = kx + my + wt$, where k, m, and w are arbitrary non-zero constants, look for traveling wave solutions of Eq. (2), and transform it to the ordinary differential equation

$$Q(U, U', U'', \ldots) = 0, \tag{3}$$

where prime denotes the derivative with respect to ζ.

Step 2. (*Simplify the nonlinear ODE*) Integrate Eq. (3), if possible, term by term one or more times. This yields constant(s) of integration. The integration constant(s) can be set to zero for simplicity.

Step 3. (*Make an ansatz*) Suppose the solution $U(\zeta)$ of Eq. (3) can be expressed in the form

$$U(\zeta) = \frac{a_c \exp(c\zeta) + \cdots + a_{-d} \exp(-d\zeta)}{b_p \exp(p\zeta) + \cdots + b_{-q} \exp(-q\zeta)}, \tag{4}$$

where c, d, p and q are unknown positive integers to be determined, a_i and b_j are unknown constants.

Step 4. (*Determine the parameters*) Determine the highest order nonlinear term and the linear term of highest order in Eq. (3) and express them in terms of (4). Then, in the resulting terms, balance the highest order Exp-function to determine c and p, and the lowest order Exp-function to determine d and q.

Step 5. (*Generate a set of algebraic equations*) Substitute (4) into Eq. (3) and equate the coefficients of $\exp(l\eta)$ to zero, obtain a *system* of algebraic equations for a_i, b_j, k, m and w. Then, to determine these constants, solve the system with the aid of a computer algebra system such as Mathematica.

Step 6. (*Obtain exact solutions*) Substitute the values solved in Step 5 into expression (4) and find the traveling wave solutions of Eq. (2). Then, ıt is necessary to substitute them into the original Eq. (2) to assure the correctness of the solutions.

3. Analytic solutions to the (2 + 1)D-ZK equation

To seek for the traveling wave solutions to the (2 + 1)D-ZK equation (1), we make the transformation $u(x,y,t) = V(\zeta)$, $\zeta = kx + my + wt$, where k, m and w are constants to be determined later. Then, integrating the resulting ODE once and setting the constant of integration to zero, we get

$$(\beta k^3 + \gamma k m^2)V'' + \frac{\alpha k}{2}V^2 + (\delta k + w)V = 0, \tag{5}$$

where primes denote the derivatives with respect to ζ. Now, we make an ansatz

$$V(\zeta) = \frac{a_c \exp(c\zeta) + \cdots + a_{-d} \exp(-d\zeta)}{b_p \exp(p\zeta) + \cdots + b_{-q} \exp(-q\zeta)} \tag{6}$$

for the solution of Eq. (5) and balance the terms V'' and V^2. By a simple calculation, we have

$$V'' = \frac{k_1 \exp[(c + 3p)\zeta] + \cdots}{k_2 \exp[4p\zeta] + \cdots} \tag{7}$$