A note on the G'/G-expansion method

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ARTICLE INFO	ABSTRACT
	We demonstrate that the G'/G-expansion method which is often used in finding exact solutions of nonlinear differential equation is equivalent to the well-known tanh-method and application of these methods gives the same exact solutions of nonlinear differential equations. © 2010 Elsevier Inc. All rights reserved.

1. Introduction

There are many different methods used for looking exact solutions of nonlinear partial differential equations. We know the inverse scattering transform [1–3] to solve the Cauchy problem for integrable partial differential equations. The Hirota method [4] is well known to obtain the solitary wave solutions.

Among non-integrable nonlinear differential equations there is a wide class of the equations that are referred to as the partially integrable. These equations have a limited number of exact solutions. To find some exact solutions of these equations the investigators usually use one of ansatz methods.

One of the typical methods in finding exact solutions of nonlinear differential equations is the tanh-method. This method was applied many times and presented widely in periodic literature. Recently a new approach for solving differential equations called the G'/G -expansion method was introduced. It has been widely used the last two years.

The aim of this letter is to demonstrate that the G'/G-expansion method and the famous tanh-method are equivalent and consequently these methods give the same solutions.

This letter is organized as follows. In Sections 2 and 3 we present the descriptions of the tanh-method and the G'/G-expansion method. In Section 4 we show that these methods are identical.

2. Description of the tanh-method

The tanh-method for finding exact solutions of nonlinear differential equations was introduced more than 20 years ago and now it is difficult to remember who was the first to use this effective approach. We should note some old publications of the application of the tanh-method to look for exact solutions of nonlinear differential equations [5–8]. Description of the tanh-method can be found in papers [9,10]. The essence of this approach is as follows:

Assume we have nonlinear partial differential equation

$$P(u, u_x, u_t, \ldots, x, t) = 0, \tag{1}$$

where P is polynomial on $u(x,t)$ and its derivatives. One can say that the method contains five steps.

Step 1. Reduction of Eq. (1) to nonlinear ordinary differential equation.

At this step using the travelling wave ansatz $u(x,t) = U(z), z = x - C_0 t$ Eq. (1) is reduced to the nonlinear ordinary differential equation (ODE)

$$E(U, U_z, U_{zz} \ldots, z) = 0. \tag{2}$$

Step 2. Hypothesis that the solution for Eq. (2) can be found in the form of the finite series on hyperbolic tangents.

At this step we suppose that exact solutions of Eq. (2) can be found in the form

$$U = \sum_{k=0}^{N} b_k \tanh^k(m(z - z_0)), \tag{3}$$

where N is integer, coefficients b_k and parameter m are unknown values that can be found after substitution (3) into (2).

Step 3. Finding positive integer N in the finite series of the solution with the hyperbolic tangents.

Substituting

$$U = b_N \tanh^N(mz), \tag{4}$$

into Eq. (2) and equating the maximal power of the hyperbolic tangent to zero we find the values N and b_N. This step corresponds to the first step of analysis of the nonlinear differential equation on the Painlevé property. Some authors say that this step is the application of the homogenous method because integer N can be found taking the balance into account between the highest order derivatives and nonlinear terms in (2).

Step 4. Determination of coefficients b_k and parameter m.

Substituting expression (3) into Eq. (2) and equating expressions of different power of $\tanh^k(m(z - z_0))$ to zero we are looking for coefficients b_k and parameter m.

Step 5. Presentation of solutions for Eq. (2).

Substituting b_k and m into formula (2) we obtain the exact solutions of nonlinear differential Eq. (2) as a result of the application of the tanh-method.

One can see that application of the tanh-method is a simple procedure for finding exact solutions of nonlinear differential equations. However many investigators want to have "new method" for finding exact solutions to find "new solutions" of nonlinear differential equations.

3. Description of the G'/G-expansion method

The G'/G-method for finding exact solutions of nonlinear differential equations was introduced in paper [11]. Currently this method is often used for searching exact solutions of nonlinear differential equations (see [12–23] for an example).

The essence of this approach can be formulated as follows. The first step of this method is equivalent to the tanh-method.

Step 1. Reduction of Eq. (1) to nonlinear ordinary differential Eq. (2).

Taking the travelling wave ansatz into account again we have the nonlinear ordinary differential equation as a result.

Step 2. Hypothesis that the solution of Eq. (2) can be searched for in the form of the finite series of the logarithmic derivative on the function $G(z)$.

At this step we assume that the exact solutions of Eq. (2) can be found in the form of the finite series on the logarithmic derivatives

$$U = \sum_{k=0}^{N} a_k \left(\frac{G'(z)}{G(z)}\right)^k, \quad G' = \frac{dG}{dz}, \tag{5}$$

where coefficients b_k can be found and $G(z)$ is a solution of the linear second-order differential equation

$$G'' - \lambda G' - \mu G = 0, \tag{6}$$

where λ and μ are constants determined as well.

Step 3. Finding positive integer N in the finite series (5) on the logarithmic derivative of the function $G(z)$.