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Application of Hankel transforms to boundary value problems of water flow due to a circular source

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ABSTRACT

With the aid of Hankel transform technique, we obtain close-form solutions for discontinuous boundary-condition problems of water flow due to a circular source, which located on the upper surface of a confined aquifer. Owing to difficult evaluations of the original solutions that are in a form of an infinite range integral with a singular point and Bessel functions in integrands, we adopt two numerical algorisms to transform the original solutions as a series form for convenient practical applications. We apply the solutions in series form to numerical examples to analyze the characteristics of the flow in the confined aquifers subjected to pumping or recharge. By numerical examples, it indicates that: the drawdown will reduce with the increase of the layer thickness and the distance from the center of a circular source when pumping in a region with a finite thickness and a finite width; two algorisms for closed-form solutions of an infinite range integral have almost the same results, but the second algorism is superior for a faster convergence; in a semi-infinite confined aquifer, the drawdown due to a constant pumping rate Q and uplift due to recharge by a given hydraulic head s_0 will both decrease with the increase of K_r/K_v; however, the radius r_0 of the circular source has a reverse influence on the drawdown and the uplift, i.e., the drawdown decrease with the increase of r_0, while the uplift increase with r_0.
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1. Introduction

In geotechnical engineering and hydraulic engineering etc, researchers and engineers concern very much about the drawdown of hydraulic head due to pumping and uplift caused by recharge in the aquifer, therefore many mathematical models and corresponding analytical solutions (see [1–4]) have been developed to analysis flows of underground fluid. As well known, there are three kinds of boundary conditions, i.e., Dirichlet (first kind), Neuman (second kind) and Robin (third kind) boundary conditions (see [5]). In the existing mathematical models solved analytically, the boundary conditions for surfaces of the domain of corresponding problems are mostly continuous. However, a number of problems in the fluid flow involve discontinuous boundary conditions, i.e., with the situation when there is an abrupt change in the type of boundary condition along surface. These can be called split boundary value problems, which are distinguished from problems with Robin (third kind) boundary conditions (see [6]). Analytical solutions of such kind problems for fluid flows are few to be discussed in the literature, so in this paper, we present three cases (two for the flow due to pumping and the other for the recharge) with axial symmetry and apply the Hankel transform to solve these problems. Because the closed-form solutions for the semi-infinite domain problem are in a form of an infinite range integral with a

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singular point and Bessel functions in integrands, they are difficult to evaluate in engineering analysis. We utilize two algorisms to attained transformed solutions in series forms that are convenient for analysis in engineering. Finally, we apply these solutions to numerical examples to analyze characteristics of the flow in the confined aquifers subjected to pumping or recharge with a circular source.

2. Definitions and summary

In this paper, the Hankel transform and the Abel transform technique are applied, so some definitions and properties are introduced as below.

Two forms of Hankel transform and inversion transform used in [7], i.e., defined in a semi-infinite domain as [8] and in a finite radial domain as [9], are written as, respectively:

$$H_v[f(r);\xi] = \int_0^\infty \xi f(r) J_v(\xi r) dr = F(\xi), \tag{1}$$

and its corresponding inversion formula is

$$H_v^{-1}[F(\xi);r] = \int_0^\infty r f(\xi) J_v(\xi r) dr = f(r). \tag{2}$$

Finite Hankel transform:

$$H_v[f(r);\xi_n] = \int_0^a \xi f(r) J_v(\xi_n r) dr = \overline{F}(\xi_n), \tag{3}$$

where $f(r)$ is a function satisfying Dirichlet conditions, and ξ_n are positive roots of the transcendental equation $J_v(\xi_n r) = 0$.

Inversion formula of (3) is

$$H_v^{-1}\left[\overline{F}(\xi_n);r\right] = \frac{2}{a^2}\sum_{n=1}^{\infty}\frac{\bar{f}(\xi_n)J_v(\xi_n a)}{\left[J_v'(a\xi_n)\right]^2} = f(r). \tag{4}$$

In many mixed boundary-condition problems solved by adoption of Hankel transform, dual equations may arise when applying Hankel transform inversion, and dual equations have a similar form as

$$\begin{cases} F(u)J_v(ru)du = G(r), & 0 < r < 1 \\ uF(u)J_v(ru)du = G(r), & r > 1 \end{cases} \tag{5}$$

where $F(u)$ needs to be determined. In this paper, Abel transform used in [10] is utilized to solve dual equations, so we introduce its definition as

$$A_1[f(x);x] = \sqrt{\frac{2}{\pi}}\int_0^x \frac{f(x)}{\sqrt{x^2 - r^2}} dr, \tag{6}$$

$$A_2[f(x);x] = \sqrt{\frac{2}{\pi}}\int_x^\infty \frac{f(x)}{\sqrt{r^2 - x^2}} dr. \tag{7}$$

The Abel Transform has the properties:

$$A_1^{-1}\left\{H_0\left[\xi^{-1}A(x);r\right];x\right\} = F_c\{A(\xi);x\}, \tag{8}$$

$$A_2\{rH_0[A(\xi);r];x\} = F_c\{A(\xi);x\}, \tag{9}$$

where A_1 and A_1^{-1} denotes the Abel transform operator and its inversion operator respectively; H_0 denotes zero order Hankel transform, and F_c means Fourier **cosine** transform defining as

$$F_c\{A(\xi);x\} = \sqrt{\frac{2}{\pi}}\int_0^\infty f(\xi)\cos(\xi x) d\xi. \tag{10}$$

3. Description of problem in confined aquifer flow due to circular source

In this paper, we will consider two types of hydraulic flow caused by the circular source on the upper surface of confined aquifer layer, which are shown in Figs. 1 and 2, respectively. The first type means pumping water from a confined aquifer, which is an engineering activity encountered frequently such as withdraw of water from a spring for industrial and domestic purposes, or lowering the water head of foundation pits for the convenience of civil engineering construction. And the second type indicates that water is injected into the confined aquifer, and in reality, it is a form of the artificial recharge that can prevent over-subsidence caused by drawdown of groundwater.

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