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Solitons and periodic solutions for the Rosenau–KdV and Rosenau–Kawahara equations

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ABSTRACT

In this work we use the sine–cosine and the tanh methods for solving the Rosenau–KdV and Rosenau–Kawahara equations. The two methods reveal solitons and periodic solutions. The study confirms the power of the two schemes.

1. Introduction

To describe the dynamics of dense discrete systems, Rosenau [1,2] proposed the following so-called Rosenau equation

$$u_t + u_{xxxxt} + u_x + uu_x = 0. \tag{1.1}$$

The existence and the uniqueness of the solution for (1.1) was proved by Park [3]. for the further consideration of the nonlinear wave, the viscous term $+u_{xxx}$ needs to be included

$$u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} = 0. \tag{1.2}$$

This equation is called Rosenau–KdV equation here. On the other hand, the viscous term $-u_{xxxxx}$ needs to be included

$$u_t + u_{xxxxt} + u_x + uu_x + u_{xxx} - u_{xxxxx} = 0. \tag{1.3}$$

This equation is called Rosenau–Kawahara equation here. The objectives of this work are twofold. Firstly, we seek to establish exact solutions for the Rosenau–KdV and Rosenau–Kawahara equations. Secondly, we aim to implement two strategies to achieve our goal, namely, the tanh method [4–7] and the sine–cosine method [8–12], and to emphasize the applicability of these methods in handling nonlinear problems.

The sine–cosine method [8–12] and the tanh method [4–7] have the advantage of reducing the nonlinear problem to a system of algebraic equations that can be easily solved by using a symbolic computation system such as Mathematica or Maple. The power of the two methods that will be used derives from the ease of use for determining shock or solitary types of solution. In what follows, the sine–cosine and the tanh methods will be reviewed briefly.

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2. The two methods

2.1. The sine–cosine method

In this section, we give a brief description of the sine–cosine method as follows. For the given a nonlinear evolution equations, say, in two variables

$$\phi(u, u_t, u_x, u_{tt}, u_{xx}, \ldots) = 0. \tag{2.1}$$

We seek the following travelling wave solutions:

$$u(x,t) = u(\xi), \quad \xi = x - ct,$$

which are of important physical significance, c is constant to be determined later. Then Eq. (2.1) reduces to nonlinear ordinary differential equations

$$\psi(u, -cu_\xi, u_\xi, c^2 u_{\xi\xi}, u_{\xi\xi}, \ldots) = 0. \tag{2.2}$$

Eq. (2.2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros. The solutions of the reduced ODE equation can be expressed in the form

$$u(\xi) = \begin{cases} \lambda \cos^\beta(\mu\xi), & |\xi| \leqslant \frac{\pi}{2\mu}, \\ 0, & \text{otherwise} \end{cases} \tag{2.3}$$

or in the form

$$u(\xi) = \begin{cases} \lambda \sin^\beta(\mu\xi), & |\xi| \leqslant \frac{\pi}{2\mu}, \\ 0, & \text{otherwise} \end{cases} \tag{2.4}$$

where λ, μ and β are parameters that will be determined, μ and c are the wavenumber and the wave speed respectively. These assumptions give

$$u'' = \lambda\beta(\beta-1)\mu^2 \cos^{\beta-2}(\mu\xi) - \lambda\beta^2\mu^2 \cos^\beta(\mu\xi), \tag{2.5}$$

and

$$u^{(iv)} = \lambda\mu^4\beta^4 \cos^\beta(\mu\xi) - 2\lambda\mu^4\beta(\beta-1)(\beta^2-2\beta+2)cos^{\beta-2}(\mu\xi) + \lambda\mu^4\beta(\beta-1)(\beta-2)(\beta-3)cos^{\beta-4}(\mu\xi), \tag{2.6}$$

where similar equations can be obtained for the sine assumption. Using the sine–cosine assumptions and its derivatives in the reduced ODE gives a trigonometric equation in $\cos^R(\mu\xi)$ or $\sin^R(\mu\xi)$ terms. The parameters are then determined by first balancing the exponents of each pair of cosines or sines to determine R. We next collect all coefficients of the same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$, where these coefficients have to vanish. This gives a system of algebraic equations in the unknowns β, λ and μ that will be determined. The solutions proposed in (2.3) and (2.4) follow immediately.

2.2. The tanh method

The tanh method is developed by Malfliet [4–6] where the tanh is used as a new variable, since all derivatives of a tanh are represented by a tanh also. Introducing a new independent variable

$$Y = tanh(\mu\xi), \tag{2.7}$$

leads to the change of derivatives

$$\begin{aligned}
\frac{d}{d\xi} &= \mu(1-Y^2)\frac{d}{dY}, \\
\frac{d^2}{d\xi^2} &= -2\mu^2 Y(1-Y^2)\frac{d}{dY} + \mu^2(1-Y^2)^2\frac{d^2}{dY^2}, \\
\frac{d^3}{d\xi^3} &= 2\mu^3(1-Y^2)(3Y^2-1)\frac{d}{dY} - 6\mu^3 Y(1-Y^2)^2\frac{d^2}{dY^2} + \mu^3(1-Y^2)^3\frac{d^3}{dY^3}, \\
\frac{d^4}{d\xi^4} &= -8\mu^4 Y(1-Y^2)(3Y^2-2)\frac{d}{dY} + 4\mu^4(1-Y^2)^2(9Y^2-2)\frac{d^2}{dY^2} \\
&\quad - 12\mu^4 Y(1-Y^2)^3\frac{d^3}{dY^3} + \mu^4(1-Y^2)^4\frac{d^4}{dY^4}.
\end{aligned} \tag{2.8}$$

We then apply the following finite series expansion:

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^k, \tag{2.9}$$