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# Robust global synchronization of complex networks with neutral-type delayed nodes

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#### ABSTRACT

In this paper, the problems of robust global exponential synchronization for a class of complex networks with time-varying delayed couplings are considered. Each node in the network is composed of a class of delayed neural networks described by a nonlinear delay differential equation of neutral-type. Since model errors commonly exist in practical applications, the parameter uncertainties are involved in the considered model. Sufficient conditions that ensure the complex networks to be robustly globally exponentially synchronized are obtained by using the Lyapunov functional method and some properties of Kronecker product. An illustrative example is presented to show the effectiveness of the proposed approach.

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### 1. Introduction

Research on recurrent neural networks (RNNs) has gained considerable attention in recent years due to their strong backgrounds in applications. It is well known that time delays exist commonly in real-world systems, which are often caused by the finite switching speeds of the amplifiers as well as traffic congestions in the process of signal transmission. Recent research reveals that time delays are frequently important sources of oscillation and instability in many dynamic systems. Therefore, much efforts have been paid in the past years for analyzing dynamic behaviors of RNNs with various types of time delays, such as constant time delays [1–3], time-varying delays [3–5], interval time delays [6,7], discrete and distributed delays [8–10]. The derived criteria are usually classified into two categories: delay-independent and delay-dependent. Delay-dependent criteria are less conservative than delay-independent ones, especially when the size of a delay is small [11–13].

More recently, in order to describe dynamics more precisely for some complicated systems, the information about derivatives of the past states has been introduced in the state equations of a considered system. This kind of RNNs is termed as neutral-type RNNs. The problem of stability for such a class of RNNs has been investigated in many references. For example, in [14], the stability problem for a class of delayed NNs of the neutral-type was studied. A delay-dependent sufficient condition, which ensures the existence and uniqueness of the equilibrium point and global exponential stability was obtained in terms of linear matrix inequalities (LMIs). The criterion was improved by proposing an LMI optimization approach in [15]. The robust stability for a class of uncertain NNs of neutral-type was investigated in [16] by using the

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Lyapunov method, and the criteria were also further improved in [17]. The global exponential stability criteria for a class of uncertain NNs of neutral-type with mixed delays were achieved in [18], by using the LMI and Razumikhin-like approaches, while in [19], the design problem of state estimator for a class of RNNs of neutral-type with interval time-vary-ing delays was addressed.

On the other hand, synchronization of an array of coupled NNs which falls to the field of complex networks has bee found a lot of potential applications in many areas, including information science, secure communication, chemical, biological systems and so on. In [20], the authors proposed "the autowave principles" for parallel image processing by using an array of cellular neural networks, for instance. In recent years, the synchronization analysis for the complex networks which compose of different kinds of NNs has emerged as a research topic of primary significance. In [21], the synchronization problems of an array of linearly coupled identical connected delayed NNs were investigated. A similar problem has been studied in [22] by using the Lyapunov method and Hermitian matrix theory. In [23], the global exponential synchronization in arrays of coupled identical delayed NNs with constant and delayed coupling was studied. The global exponential synchronization of coupled connected RNNs with both discrete and distributed delays was investigated in [24] by using the LMI and Kronecker product technique. In [25], the synchronization problem of stochastic delayed discrete-time complex networks was studied by using an LMI optimization approach. As far as the neutral-type NNs are concerned, the synchronization control for a kind of master-response setup was studied in [26]. The research work has been further extended to the case of neutral-type NNs with stochastic perturbation in [27]. However, the synchronization problem for an array of neutral-type NNs has not been investigated. Due to the fact that many real-world delayed systems can be described by neutral-type NNs, the investigation on the robust global synchronization problem can have promising applications in a variety of areas, such as reliable multiple robots formulation [28], chaotic secure communication [29].

In this paper, we consider the robust global synchronization problem of a class of complex networks with time-varying delayed couplings. Each node in the network is composed of a class of delayed NNs described by a nonlinear delay differential equation of neutral-type. Two cases of the time delays are considered in this paper. One is that the time delays are bounded and their derivatives are less than one but not less than zero. In the other case, the time delays are only required to be bounded, in which very fast continuous changes in the time delays are allowed. Sufficient conditions that ensure the complex networks globally robustly synchronized are obtained based on the Lyapunov functional method and Kronecker product techniques. A numerical example is presented to show the effectiveness and applicability of the proposed approach.

*Notation:*  $\mathbb{R}^n$  denotes the *n*-dimensional Euclidean space.  $\mathbb{R}^{n \times m}$  is the set of real  $n \times m$  matrices. *I* is the identity matrix of appropriate dimensions.  $\|\cdot\|$  stands for the Euclidean vector norm or spectral norm as appropriate, and diag{·} stands for a block-diagonal matrix. The notation X > 0 (respectively, X < 0), for  $X \in \mathbb{R}^{n \times n}$  means that the matrix X is a real symmetric positive definite (respectively, negative definite). The asterisk \* in a matrix is used to denote term that is induced by symmetry.  $\mathbb{E}\{\cdot\}$  denotes the expectation. The Kronecker product of matrices Q and R is denoted as  $Q \otimes R$ . We let  $\tau > 0$  and  $\mathscr{C}([-\tau, 0]; \mathbb{R}^n)$  denote the family of continuous functions  $\phi$  from  $[\tau, 0]$  to  $\mathbb{R}^n$  with the norm  $|\phi| = \sup_{-\pi \leqslant 0 \le 0} ||\phi(\theta)||$ .

#### 2. Problem formulation

We consider the following array of identical delayed NNs for i = 1, 2, ..., n:

$$\dot{x}_{i}(t) - C\dot{x}_{i}(t - \tau_{1}(t)) = A_{t}x_{i}(t) + B_{t}f(x_{i}(t)) + W_{t}f(x_{i}(t - \tau_{2}(t))) + \sum_{j=1}^{n} g_{ij}\Gamma_{t}x_{j}(t) + \sum_{j=1}^{n} h_{ij}\Upsilon_{t}x_{j}(t - \tau_{3}(t)),$$
(1)

where  $x_i(t) = [x_{i1}(t), x_{i2}(t), \dots, x_{im}(t)]^T \in \mathbb{R}^m$  is the state vector of the *i*th node in the networks at time t.  $G = (g_{ij})_{n \times n}$  and  $H = (h_{ij})_{n \times n}$  are the out-coupling configuration matrices representing the coupling strength and the topological structure of the complex networks;  $\tau_k(t)$  (k = 1, 2, 3) are scalars which denote the time delays;  $f(x_i(t)) = [f_1(x_{i1}(t)), \dots, f_m(x_{im}(t))]^T$  is an unknown but sector-bounded nonlinear function. In system (1),

$$A_t = A + \Delta A(t), \quad B_t = B + \Delta B(t), \quad W_t = W + \Delta W(t),$$
  

$$\Gamma_t = \Gamma + \Delta \Gamma(t), \quad \Upsilon_t = \Upsilon + \Delta \Upsilon(t),$$
(2)

where *A* denotes a known connection matrix. *B* and *W* denote, respectively, the connection weight matrix and the delayed connection weight matrix.  $\Gamma$ ,  $\Upsilon \in \mathbb{R}^{m \times m}$  are matrices describing the inner-coupling between the subsystems at time *t* and  $t - \tau_3(t)$ , respectively.  $\Delta A(t)$ ,  $\Delta B(t)$ ,  $\Delta W(t)$ ,  $\Delta \Gamma(t)$  and  $\Delta \Upsilon(t)$  represent the parameter uncertainties of the system, which are assumed to be of the form

$$[\Delta A(t), \Delta B(t), \Delta W(t), \Delta \Gamma(t), \Delta \Upsilon(t)] = MF(t)[E_a, E_b, E_w, E_\gamma, E_v].$$
(3)

The initial conditions associated with (1) are

$$x_{ij}(s) = \varphi_{ij}(s) \in \mathscr{C}([-\rho, 0], \mathbb{R}), \quad i = 1, 2, \dots, n, \ j = 1, 2, \dots, m,$$
(4)

where  $\rho$  is the maximum value of  $\tau_k(t)$  (k = 1, 2, 3).

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