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Exact solutions of the Drinfel'd–Sokolov–Wilson equation using the Exp-function method

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ABSTRACT

The generalized solitary solutions of the classical Drinfel'd–Sokolov–Wilson equation (DSWE) are obtained using the Exp-function method. Then, some of these solutions are easily converted into kink-shaped solutions and blow-up solutions by a simple transformation.

1. Introduction

The investigation of exact solutions of nonlinear wave equations plays an important role in the study of nonlinear physical phenomena. Recently, many effective methods for obtaining exact solutions of nonlinear wave equations have been proposed, such as Bäcklund transformation method [1], homogeneous balance method [2,3], bifurcation method [4–6], the hyperbolic tangent function expansion method [7,8], the Jacobi elliptic function expansion method [9–11], Hirotas bilinear method [12] and so on.

In 2006, He and Wu proposed a new method, called Exp-function method, to obtain exact solutions of nonlinear differential equations [13]. Then, it was studied in a lot of problems [14–17] and so on. On the other hand, the authors in the paper [18] analyze the application of the Exp-function method to some nonlinear evolution equations. Recently, possibilities of some new methods in mathematical physics have been discussed by the authors.

In this paper, we consider the classical Drinfel'd–Sokolov–Wilson equation

$$u_t + pvv_x = 0, \tag{1}$$

$$v_t + qv_{xxx} + ruv_x + su_x v = 0. \tag{2}$$

where p, q, r, s are some nonzero parameters.

Recently, DSWE and the coupled DSWE, a special case of the classical DSWE, have been studied by several authors [19–25]. In this study, we construct the Exp-function method to solve Eqs. (1) and (2). The solution procedure of this method, by the help of symbolic computation of Matlab or Mathematica, is of utter simplicity.

2. Exp-function method

Using a complex variation η defined as $\eta = k(x - ct)$, we can convert Eqs. (1) and (2) into ordinary different equations, which read

$$-cu' + pvv' = 0, \tag{3}$$

$$-cv' + qk^2 v''' + ruv' + su'v = 0, \tag{4}$$

where the prime denotes the derivative with respect to η.

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Integrating Eq. (3), we obtain

$$u = \frac{pv^2}{2c} + c_1, \tag{5}$$

where c_1 is an integration constant.

Substituting u into Eq. (4) yields:

$$2cqk^2 v''' + p(r+2s)v^2 v' + 2c(rc_1 - c)v' = 0. \tag{6}$$

According to the Exp-function method, we assume that the solution of Eq. (6) can be expressed in the following form:

$$v(\eta) = \frac{\sum_{n=-c}^{d} a_n \exp(n\eta)}{\sum_{m=-p}^{q} b_m \exp(m\eta)}, \tag{7}$$

where c, d, p and q are positive integers which are unknown to be further determined, a_n and b_m are unknown constants. Eq. (7) can be rewritten in an alternative form as follows:

$$v(\eta) = \frac{a_c \exp(c\eta) + \cdots + a_{-d} \exp(-d\eta)}{b_p \exp(p\eta) + \cdots + b_{-q} \exp(-q\eta)}. \tag{8}$$

To determine values of c and p, we balance the linear term of highest order of Eq. (6) with the highest order nonlinear term. By simple calculation, we have

$$v''' = \frac{c_1 \exp[(7p+c)\eta] + \cdots}{c_2 \exp[8p\eta] + \cdots} \tag{9}$$

and

$$v^2 v' = \frac{c_3 \exp[(p+3c)\eta] + \cdots}{c_4 \exp[4p\eta] + \cdots} = \frac{c_3 \exp[(5p+3c)\eta] + \cdots}{c_4 \exp[8p\eta] + \cdots}, \tag{10}$$

where c_i are determined coefficients only for simplicity.

Balancing highest order of Exp-function in Eqs. (9) and (10), we have

$$7p + c = 5p + 3c \tag{11}$$

which leads to the result

$$p = c. \tag{12}$$

Similarly to determine values of d and q, we balance the linear term of lowest order in Eq. (6) with the lowest order nonlinear term v''' and $v^2 v'$, we have

$$v''' = \frac{\cdots + d_1 \exp[-(7q+d)\eta]}{\cdots + d_2 \exp[-8q\eta]} \tag{13}$$

and

$$v^2 v' = \frac{\cdots + d_3 \exp[-(q+3d)\eta]}{\cdots + d_4 \exp[-4q\eta]} = \frac{c_3 \exp[-(5q+3d)\eta] + \cdots}{c_4 \exp[-8q\eta] + \cdots}, \tag{14}$$

where d_i are determined coefficients only for simplicity.

Balancing lowest order of Exp-function in Eqs. (13) and (14), we have

$$-(7q+d) = -(5q+3d) \tag{15}$$

which leads to the result

$$q = d. \tag{16}$$

For simplicity, we set $p = c = 1$ and $q = d = 1$, then Eq. (8) reduces to

$$v(x,t) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)}. \tag{17}$$

In case $b_1 \neq 0$ Eq. (17) can be simplified as

$$v(x,t) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{\exp(\eta) + b_0 + b_{-1} \exp(-\eta)}. \tag{18}$$

Substituting Eq. (18) into Eq. (6), we have

$$\frac{1}{A}[C_3 \exp(3\eta) + \cdots + C_0 + \cdots + C_{-3} \exp(-3\eta)], \tag{19}$$