Applied Mathematics and Computation 217 (2011) 4900–4910

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Uniform persistence for a two-species ratio-dependent predator–prey system with diffusion and variable time delays ☆

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ARTICLE INFO

Keywords:
Variable time delay
Diffusion
Uniform persistence

ABSTRACT

A two-species ratio-dependent predator–prey diffusion model with variable time delays is investigated. By constructing some suitable Lyapunov functionals and utilizing some analysis techniques, we obtain the permanence of this system. Our results generalize and improve the results of [10,11].

1. Introduction

Because of the ecological effects of human activities and industry, e.g., the location of manufacturing industries and pollution of the atmosphere, rivers and soil, more and more habitats have been broken into patches and some of the patches have been polluted. This environment endangers population survival and variety. To prevent some small animals from being predated, people often construct some barriers to help them. The important and most basic ecological problem in the study of population dynamics concerns the permanence and extinction.

Since the pioneering theoretical work by Skellem [1], many works have focused on the effect of spatial factors which play a crucial rule in the persistence and stability of population [2–12]. In fact, dispersal between patches often occurs in ecological environments, and more realistic models should include the dispersal process.

Recently, many authors have studied the permanence and stability of Lotka–Volterra diffusion models [3–8]. They assume that the per capita rate of predation depends on the prey numbers only. According to numerous fields and laboratory experiments and observations [10–13], a more suitable general predator–prey theory should be based on the so-called ratio-dependent theory, which can be roughly stated as that the per capita predator growth rate should be a function of the ratio of prey to predator abundance, and so should be the so-called predator functional response. Generally, a ratio-dependent predator–prey model takes the form:

$$\begin{cases} x' = xf(x) - yp\left(\frac{x}{y}\right), \\ y' = cyq\left(\frac{x}{y}\right) - dy, \end{cases} \tag{1.1}$$

here $p(x)$ is the so-called predator functional response. Particularly, the ratio-dependent type predator–prey model with Michaelis–Menten type functional response is as follows:

$$\begin{cases} x' = ax\left(1 - \frac{x}{K}\right) - \frac{cxy}{my+x}, \\ y' = y\left(-d + \frac{fx}{my+x}\right), \end{cases} \tag{1.2}$$

where $\frac{cu}{m+u}$ is a Michaelis–Menten type functional response function.

☆ Supported by the Natural Science Foundation of Overseas Chinese Affairs of the State Council (No. 09QZR10) and (No. 07QZR09).
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doi:10.1016/j.amc.2010.11.031

In addition, it is generally recognized that some kind of time delays are inevitable in population interactions. Thus, a model includes not only the dispersal but also some past states of the species. And so, in [10], Xu and Chen studied the following delayed system.

$$\begin{cases} x_1'(t) = x_1(t)\left[r_1 - a_1x_1(t) - \frac{c_1x_3(t)}{x_1(t)+px_3(t)}\right] + D_1[x_2(t) - x_1(t)], \\ x_2'(t) = x_2(t)[r_2 - a_2x_2(t)] + D_2[x_1(t) - x_2(t)], \\ x_3'(t) = x_3(t)\left[-r_3 + \frac{c_2x_1(t-\tau)}{x_1(t-\tau)+px_3(t-\tau)}\right], \end{cases} \tag{1.3}$$

where $x_i(t)$ represents the prey population in the ith patch, $i = 1, 2$; and $x_3(t)$ represents the predator population. $\tau > 0$ is a constant delay due to gestation. D_i is a positive constant and denotes the dispersal rate, $i = 1, 2$. $a_i(i = 1,2)$ and m are positive constants. The authors obtained some sufficient conditions for uniform persistence and stability of system (1.3). In [11], Gou and Pu studied the following nonautonomous delayed system.

$$\begin{cases} x_1'(t) = x_1(t)\left[r_1(t) - a_1(t)x_1(t) - \frac{c_1(t)x_3(t)}{x_1(t)+p(t)x_3(t)}\right] + D_1(t)[x_2(t) - x_1(t)], \\ x_2'(t) = x_2(t)[r_2(t) - a_2(t)x_2(t)] + D_2(t)[x_1(t) - x_2(t)], \\ x_3'(t) = x_3(t)\left[-r_3(t) + \frac{c_2(t)x_1(t-\sigma)}{x_1(t-\sigma)+p(t)x_3(t-\sigma)}\right], \end{cases} \tag{1.4}$$

where $x_i(t)$ represents the prey population in the ith patch, $i = 1, 2$; and $x_3(t)$ represents the predator population. $\tau > 0$ is a constant delay due to gestation. $D_i(t)$ is a continuous function and denotes the dispersal rate, $i = 1, 2$. $a_i(t)(i = 1,2)$ and $m(t)$ are continuous functions. The authors proved that system (1.4) is uniform persistence under some appropriate conditions.

However, to this day, few scholars consider the influence of the diffusion on nonlinear predator–prey system with variable time delays and increasing rate of species.

Stimulated by the works of [10,11,14], we consider the following two species ratio-dependent predator–prey model with diffusion, variable time delays and Michaelis–Menten type functional response

$$\begin{cases} x_1'(t) = x_1(t)[r_1(t) - a_1(t)x_1(t) - b_1(t)x_1(t-\tau_1(t)) - \zeta(t)] + D_1(t)[x_2(t) - x_1(t)], \\ x_2'(t) = x_2(t)[r_2(t) - a_2(t)x_2(t) - b_2(t)x_2(t-\tau_2(t))] + D_2(t)[x_1(t) - x_2(t)], \\ x_3'(t) = x_3(t)\left[-r_3(t) + \frac{c_2(t)x_1(t-\sigma_2(t))}{p_3(t)x_1(t-\sigma_2(t))+p_4(t)x_3(t-\sigma_2(t))}\right], \end{cases} \tag{1.5}$$

where $\zeta(t) = \frac{c_1(t)x_3(t-\sigma_1(t))}{p_1(t)x_1(t-\sigma_1(t))+p_2(t)x_3(t-\sigma_1(t))}$, $x_1(t)$ and $x_3(t)$ are the densities of species $X_1$and X_3 in patch 1 at the time t, respectively; $x_2(t)$ is the density of specie X_1 in patch 2 at the time t; the species X_1 is the prey of species X_3; $D_i(t)(i = 1,2)$ are diffusion coefficients of species X_1. Species X_3 is confined to patch 1 while species X_1 can diffuse between two patches. $b_i(t)x_i(t-\tau_i(t))(i = 1,2)$ represent the influence of the past state of species X_1 in the ith patch at the time t, respectively. For more backgrounds and biological adjustment, one refers to [10,11,14], and the references cited therein.

Let $f^L = \inf\{f(t)\colon t \in R_+\}$, $f^M = \sup\{f(t)\colon t \in R_+\}$, if $f(t)$ is a bounded continuous function.

In this paper, for system (1.5) we always assume that

(H_1) all the functions and time delays are continuous, bounded functions on $[0,+\infty)$, and satisfy

$$\begin{aligned} &0 < a_j^L \leqslant a_j(t) \leqslant a_j^M, \quad 0 \leqslant b_j^L \leqslant b_j(t) \leqslant b_j^M, \quad 0 \leqslant \tau_j^L \leqslant \tau_j(t) \leqslant \tau_j^M, \\ &0 < c_j^L \leqslant c_j(t) \leqslant c_j^M, \quad 0 \leqslant D_j^L \leqslant D_j(t) \leqslant D_j^M, \quad 0 \leqslant \sigma_j^L \leqslant \sigma_j(t) \leqslant \sigma_j^M, \ j = 1,2; \\ &0 < p_k^L \leqslant p_k(t) \leqslant p_k^M, \quad k = 1,2,3,4; \ 0 < r_i^L \leqslant r(t) \leqslant r_i^M, \ i = 1,2,3 \end{aligned}$$

for any $t \in [0,+\infty)$.

The aim of this paper is as follows: obtaining sufficient condition which guarantees the uniform persistence of system (1.5). We arrange our paper as follows: in the next section, we describe some background concepts and prove one lemma; in Section 3, we obtain the sufficient condition that guarantees the bound of the solutions for system (1.5); in Section 4, we prove the uniform persistence of system (1.5); in Section 5, we give two examples to illustrate the generality of Theorems 4.1 and 4.2. In particular, our results generalize and improve the results of [10,11], respectively.

2. Preliminaries and one lemma

Because of the real meaning of system (1.5), we assume that solutions of system (1.5) satisfy the initial condition

$$x_i(\theta) = \phi_i(\theta), \quad \theta \in [-\tau, 0], \ i = 1,2,3, \tag{2.1}$$

where $\tau = \max\{\tau_1^M, \tau_2^M, \sigma_1^M, \sigma_2^M\}$, and ϕ_i $(i = 1,2,3)$ are all nonnegative and bounded continuous functions on $\theta \in [-\tau,0]$ with $\phi_i(0) > 0$ (i = 1,2,3). It is well known that by the fundamental theory of functional differential equations [15], system (1.5) with the initial condition (2.1) has a unique solution

$$x(t,\phi) = (x_1(t,\phi_1), x_2(t,\phi_2), x_3(t,\phi_3))^T.$$