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Adaptive complete synchronization of the noise-perturbed two bi-directionally coupled chaotic systems with time-delay and unknown parametric mismatch

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ABSTRACT

In this paper, we design an adaptive-feedback controller to synchronize a class of noise-perturbed two bi-directionally coupled chaotic systems with time-delay and unknown parametric mismatch. Based on invariance principle of stochastic time-delay differential equations, some sufficient conditions of adaptive complete synchronization are given. Comparing with other papers, here we consider the effect of internal noise, time-delay and parametric mismatch in the synchronized process. As the illustrative examples, the famous Lorenz system and Rössler system are considered here. In order to validate the proposed scheme, numerical simulations are performed, and the numerical results show that our scheme is very effective.

1. Introduction

Synchronization of oscillations is an omnipresent phenomenon in natural systems and artificial oscillators, which has been well known to scientists, since the historical observation of phenomenon in coupled pendulum clocks [1] by Huygens. In the classical sense, synchronization means the adjustment or entrainment of the frequency of periodic oscillators by means of week interaction [2]. This phenomenon has been well studied and applied in electrical and mechanical engineering. With recent widespread studies of chaotic systems, the notion of synchronization has been extended to the chaotic system. It is well known that the sensitivity to initial condition is generic character of chaotic oscillators. Two chaotic orbits, starting from slightly different initial points, separate exponentially with time. Due to this generic feature of chaotic systems, the chaotic synchronization is deem to have a great amount of application areas, such as physics [3], biology [4], signal engineering [5], chemistry [6] and ecological science [7]. In recent years, chaotic synchronization has been studied extensively by many scientists, and various modern control methods have been proposed to synchronize chaotic systems, such as back-stepping design [8], linear feedback control [9], nonlinear control [10], adaptive control [11], active control [12], or adaptive active control [13]. However, most of above methods are just proposed for uni-directionally coupled identical systems, and did not consider the effect of noise in the synchronized process.

In fact, many systems should be described by bi-directionally coupled systems, and the synchronization between bi-directionally coupled systems is also an interesting topic. The chaotic systems are inevitably exposed to an environment which may cause their parameters a little different, and it is also difficult to estimate this parameters mismatch exactly. In practice, we should consider the effect of parametric mismatch in the synchronized process. It is well known that the mixing of useful signal and noise takes place during transfer of the information through communication channel, so

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synchronization is unavoidably subject to internal and external noise. In the past two decades, many scientists were interested in investigation of noise's effect on chaotic synchronization, and different roles of noise in synchronization were reported [14–16]. However, most of results are acquired by numerical simulations. Recently, in [17] some sufficient conditions for complete synchronization of noise-perturbed coupled chaotic systems were established, and a good method was provided to study the chaotic system perturbed by internal noise.

Motivated by above analysis, in this paper we will design an adaptive controller to synchronize the noise-perturbed two bi-directionally coupled chaotic systems with time-delayed and unknown parametric mismatch. Moreover, based on the so-called LaSalle-type invariance principle for stochastic differential equation proposed by Mao in [18], the sufficient conditions of the complete synchronization via this control scheme are given. The rest of paper is organized as follows. In Section 2, an adaptive-feedback controller is designed, and the criterion for complete synchronization is established. In Section 3, in order to validate the proposed synchronization scheme, the famous Lorenz system and Rössler are used as illustrative examples. Finally, some conclusions are written in Section 4.

2. Adaptive synchronization scheme

We begin with considering an n-dimensional chaotic system in the form of

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{p}), \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \dots, x_n)^T \in R^n$ is the state vector, $\boldsymbol{F} = (F_1, F_2, \dots, F_n)^T : R^n \to R^n$ is continuous nonlinear vector function. In this paper, we assume $\boldsymbol{F}(\boldsymbol{x}, \boldsymbol{p}^i)$ can be written as the following form:

$$\boldsymbol{F}(\boldsymbol{x}, \boldsymbol{p}) = \boldsymbol{c}(\boldsymbol{x}) + \boldsymbol{p}\boldsymbol{f}(\boldsymbol{x}). \tag{2}$$

Here $\boldsymbol{c}(\boldsymbol{x}) = (c_1(\boldsymbol{x}), c_2(\boldsymbol{x}), \dots, c_n(\boldsymbol{x}))^T$ and $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \dots, f_l(\boldsymbol{x}))^T$ are some smooth vector functions, and $\boldsymbol{p} = (p_{kj})_{n\times l}$ $(i = 1, 2, \dots, m)$ is an $n \times l$ matrixes which describe unknown parameters. One may easily check that the class of system in the form of (1) and (2) includes almost all famous chaotic systems such as Lorenz system, Rössler system and Chen's system.

Consider parametric mismatch, time delay of the signal transmission and the mixing of useful signal and noise, two bi-directionally coupled n-dimensional chaotic can be described as

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{p}^1) + \boldsymbol{C}(\boldsymbol{y}_\tau - \boldsymbol{x}) + \sigma_1(\boldsymbol{y}_\tau - \boldsymbol{x}_\tau)\dot{\boldsymbol{W}}_1, \\ \dot{\boldsymbol{y}} &= \boldsymbol{F}(\boldsymbol{y}, \boldsymbol{p}^2) + \boldsymbol{C}(\boldsymbol{x}_\tau - \boldsymbol{y}) - \sigma_2(\boldsymbol{y}_\tau - \boldsymbol{x}_\tau)\dot{\boldsymbol{W}}_2, \end{aligned} \tag{3}$$

where $\boldsymbol{x} = (x_1, x_2, \dots, x_n)^T, \boldsymbol{y} = (y_1, y_2, \dots, y_n)^T \in R^n$ are the state vectors. $\boldsymbol{y}_\tau = \boldsymbol{y}(t-\tau), \boldsymbol{x}_\tau = \boldsymbol{x}(t-\tau)$ are delayed states, and τ represents the time delay. $\boldsymbol{C}$ is an $n \times n$ real coupled matrix, which is a matrix of each node's variables used in the coupling. $[\dot{\boldsymbol{W}}_1^T, \dot{\boldsymbol{W}}_2^T] = [\eta_1(t), \eta_2(t), \dots, \eta_{2m}(t)]$ is a $2m$-dimensional white noise, in which every two elements is statistically independent, i.e., $E[\eta_i] = 0$, $E[\eta_i(t)\eta_j(t')] = \delta_{ij}\delta(t-t')$ $(i,j = 1, 2, \dots, 2m)$. $\boldsymbol{\sigma}_i = (\sigma_{i1}, \sigma_{i2}, \dots, \sigma_{im}) : R^n \to R^{n\times m}$ $(i = 1, 2)$ are continuous nonlinear matrix-valued functions, which are usually called the noise coupling strength functions.

Remark 1. The information flowing in coupled systems is not generally instantaneous. On the contrary, the finite speed of signal transmission over a distance gives rise to a finite time delay. So in bi-directionally coupled system (3), we consider the time-delay coupling. Obviously, the noise terms considered here are of multiplication case which can be interpreted in the sense of *Itô*. This kind of noise can be regarded as a result from the internal error.

In fact, the coupled systems are inevitably affected by different environment, so it is difficult to reach synchronization for the real-world coupled systems. In this paper, our aim is to design a simple controller to realize synchronization of coupled system (3), so we add the controller $\boldsymbol{u} = (u_1, u_2, \dots, u_n)^T \in R^n$ to the second subsystem

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{p}^1) + \boldsymbol{C}(\boldsymbol{y}_\tau - \boldsymbol{x}) + \boldsymbol{\sigma}_1(\boldsymbol{y}_\tau - \boldsymbol{x}_\tau)\dot{\boldsymbol{W}}_1, \\ \dot{\boldsymbol{y}} &= \boldsymbol{F}(\boldsymbol{y}, \boldsymbol{p}^2) + \boldsymbol{C}(\boldsymbol{x}_\tau - \boldsymbol{y}) - \boldsymbol{\sigma}_2(\boldsymbol{y}_\tau - \boldsymbol{x}_\tau)\dot{\boldsymbol{W}}_2 + \boldsymbol{u}. \end{aligned} \tag{4}$$

If we define the synchronized error $\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{x}$, the equation of synchronization error can be expressed as

$$\dot{\boldsymbol{e}} = \boldsymbol{F}(\boldsymbol{y}, \boldsymbol{p}^1) - \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{p}^2) - \boldsymbol{C}(\boldsymbol{y} - \boldsymbol{x}) - \boldsymbol{C}(\boldsymbol{y}_\tau - \boldsymbol{x}_\tau) - \boldsymbol{\sigma}_1(\boldsymbol{y}_\tau - \boldsymbol{x}_\tau)\dot{\boldsymbol{W}}_1 - \boldsymbol{\sigma}_2(\boldsymbol{y}_\tau - \boldsymbol{x}_\tau)\dot{\boldsymbol{W}}_2 + \boldsymbol{u}, \tag{5}$$

where $\boldsymbol{e} = (e_1, e_2, \dots, e_n)^T \in R^n$ is a n-dimensional vector. The aim of synchronization is to make $\lim_{t\to\infty}\|\boldsymbol{e}(t)\| = 0$ with probability one, i.e., $P(\lim_{t\to\infty}\|\boldsymbol{e}(t)\| = 0) = 1$. Here, $\|\cdot\|$ simply stands for the Euclidean norm.

By introducing the error vector $\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{x}$, the problem of synchronization in the coupled system (4) can be translated into a problem of how to realize the asymptotical stabilization of error system (5). So our aim is to design a simple controller u to make the error system (5) asymptotically stable at origin with probability one.

Inspired by Huang [11], we introduce the following simple adaptive-feedback controller $\boldsymbol{u}$:

$$u_i = \varepsilon_i(y_i - x_i) - \boldsymbol{q}_i\boldsymbol{f}(\boldsymbol{y}), \quad i = 1, 2, \dots, n, \tag{6}$$