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Impulsive control for a class of neural networks with bounded and unbounded delays

Ivanka M. Stamova [a,*], Rajcho Ilarionov [b], Rositsa Vaneva [b]

[a] Bourgas Free University, 8000 Bourgas, Bulgaria
[b] Technical University of Gabrovo, 5300 Gabrovo, Bulgaria

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ABSTRACT

In this paper, we study the problem of global exponential stability for a class of impulsive neural networks with bounded and unbounded delays and fixed moments of impulsive effect. We establish stability criteria by employing Lyapunov functions and Razumikhin technique. An illustrative example is given to demonstrate the effectiveness of the obtained results.

1. Introduction

In 1988, Chua and Yang [9,10] introduced a model of cellular neural networks, which has been found an important model in some fields, such as signal processing, pattern recognition, optimization and associative memories [4,5,8,13].

One of the most investigated problems in the study of neural networks is the global asymptotic stability of the equilibrium point. If an equilibrium of a neural network is globally asymptotically stable, it means that the domain of attraction of the equilibrium point is the whole space and the convergence is in real time. This is significant both theoretically and practically. Such neural networks are known to be well-suited for solving some class of optimization problems. In fact, a globally asymptotically stable neural network is guaranteed to compute the global optimal solution independently of the initial condition, which in turn implies that the network is devoid of spurious suboptimal responses.

However, in many neural network applications, such as associative memories, the network is designed so that stable equilibrium points represent stored information [14]. In this framework, relevant information is retrived by initializing the network at a point within the basin of attraction of the corresponding stable equilibrium point, and allowing the system to evolve to its stationary state. This approach to information processing has motivated studies on the asymptotic behavior of neural networks and conditions have been derived to ensure that all or almost all trajectories eventually converge to equilibria, thus avoiding spurious undamped oscillations [6,15,17,19]. Networks that satisfy such conditions are referred to as convergent or alost convergent.

Usually, constant fixed time delays in the models of delayed feedback systems serve as good approximation in simple circuits having a small number of cells. Though delays arise frequently in practical applications, it is difficult to measure them precisely. In most situations, delays are variable, and in fact unbounded. That is, the entire history affects the present. Such delay terms, more suitable to practical neural nets, are called unbounded delays. Gopalsamy and He [12] studied the system of integro-differential equations as a model for Hopfield-type neural networks involving unbounded time delay arising from the signal propagation. Some results on the stability of neural networks involving unbounded time delays are given in

* Corresponding author.
E-mail address: stamova@bfu.bg (I.M. Stamova).

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[7,11,25,26]. Few papers consider neural networks involving both variable and unbounded delay. See, for example, [27] and the references therein.

The state of electronic networks is often subject to instantaneous perturbations and experience abrupt changes at certain instants, which may be caused by switching phenomenon, frequency change or other sudden noise, that exhibit impulsive effects [1–3,20,21,24]. Impulses can make unstable systems stable so they have been widely used in many fields such as physics, chemistry, biology, population dynamics, and industrial robotics.

In the paper, we will study the exponential stability of neural networks model, which contains both variable and unbounded delays and at fixed moments of time it is subject to short-term perturbations. To our knowledge, this is the first paper studying the exponential stability of neural networks with variable and unbounded delay and impulsive effects. By using of piecewise continuous Lyapunov functions [16] and the Razumikhin technique [18,22,23] we establish criteria for global exponential stability. The conditions are independent of the form of specific delays and have important significance in both theory and applications. An example is given to demonstrate the effectiveness of the results.

2. Statement of the problem. Preliminary notes and definitions

Let $R_+ = [0, \infty)$, $t_0 \in R_+$, R^n denote the n-dimensional Euclidean space, and let $\|x\| = \sum_{i=1}^{n}|x_i|$ define the norm of $x \in R^n$. We consider the following impulsive nonautonomous cellular neural network with bounded and unbounded delays

$$\begin{cases} \dot{x}_i(t) = -d_i(t)x_i(t) + \sum\limits_{j=1}^{n} a_{ij}f_j\big(x_j(t)\big) + \sum\limits_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_j(t))) + \sum\limits_{j=1}^{n} c_{ij}\int_{-\infty}^{t} m_j(t,s)f_j(x_j(s))ds + I_i, \quad t \neq t_k,\ t \geqslant t_0, \\ \Delta x_i(t_k) = x_i(t_k+0) - x_i(t_k) = P_{ik}(x_i(t_k)), \quad k = 1,2,\ldots, \end{cases} \tag{2.1}$$

where $i = 1, 2, \ldots, n$; $x_i(t)$ corresponds to the state of the ith unit at time t; $f_j(x_j(t))$ is the activation function of the jth unit at time t; $A_{n\times n} = (a_{ij})_{n\times n}$, $B_{n\times n} = (b_{ij})_{n\times n}$, $C_{n\times n} = (c_{ij})_{n\times n}$ denote the connection weight matrices; $0 \leqslant \tau_j(t) \leqslant \tau$; I_i is the external bias on the ith neuron; $d_i(t)$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs; the delay kernel $m_j(t, s) = m_j(t-s)$, $(j = 1, 2, \ldots, n)$ is of convolution type; t_k, $k = 1, 2, \ldots$ are the moments of impulsive perturbations and satisfy $t_0 < t_1 < t_2 < \ldots$ and $\lim_{k\to\infty} t_k = \infty$. The numbers $x_i(t_k) = x_i(t_k - 0)$ and $x_i(t_k + 0)$ are, respectively, the states of the ith unit before and after impulse perturbation at the moment t_k and the functions $P_{ik}(x_i(t_k))$ represents the abrupt change of the state $x_i(t)$ at the impulsive moment t_k.

Let $J \subset R$ be an interval. Define the following classes of functions :

$PC[J, R^n] = \{\sigma : J \to R^n :\ \sigma(t)$ is continuous everywhere except at some points $t_k \in J$ at which $\sigma(t_k - 0)$ and $\sigma(t_k + 0)$ exist and $\sigma(t_k - 0) = \sigma(t_k)\}$;

$CB[J, R^n]$=$\{\sigma \in C[J, R^n]$:$\sigma(t)$ is bounded on J$\}$.

Let $\varphi \in CB[(-\infty, 0], R^n]$. Denote by $x(t) = x(t; t_0, \varphi)$, $x \in R^n$, the solution of system (2.1), satisfying the initial conditions

$$\begin{cases} x(t; t_0, \varphi) = \varphi(t - t_0), \quad -\infty < t \leqslant> t_0, \\ x(t_0 + 0; t_0, \varphi) = \varphi(0). \end{cases} \tag{2.2}$$

The solution $x(t) = x(t; t_0, \varphi) \in R^n$ of problem (2.1) (2.2) is a piecewise continuous function with points of discontinuity of the first kind t_k, $k = 1, 2, \ldots$, where it is continuous from the left, i.e. the following relations are valid

$$x_i(t_k - 0) = x_i(t_k), \quad x_i(t_k + 0) = x_i(t_k) + P_{ik}(x_i(t_k)), \quad i = 1,2,\ldots,n,\ k = 1,2,\ldots.$$

Let $\|\varphi\|_\infty = \max_{s\in(-\infty,t_0]}\|\varphi(t - t_0)\|$ be the norm of the function $\varphi \in\ CB[(-\infty, 0], R^n]$.

We introduce the following conditions:

H2.1. There exist constants $L_i > 0$ such that

$$|f_i(u) - f_i(v)| \leqslant L_i|u - v|$$

for all $u,\ v \in R,\ i = 1, 2, \ldots, n$.

H2.2. There exist constants $M_i > 0$ such that for all $u \in R$ and $i = 1, 2, \ldots, n$

$$|f_i(u)| \leqslant M_i < \infty.$$

H2.3. The delay kernel $m_i : R^2 \to R_+$ is continuous, and there exist positive numbers μ_i such that

$$\int_{-\infty}^{t} m_i(t,s)ds \leqslant \mu_i < \infty$$

for all $t \geqslant t_0,\ t \neq t_k,\ k = 1, 2, \ldots$ and $i = 1, 2, \ldots, n$.

H2.4. The functions P_{ik} are continuous on $R,\ i = 1, 2, \ldots, n,\ k = 1, 2, \ldots.$