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New exact kink solutions, solitons and periodic form solutions for a combined KdV and Schwarzian KdV equation

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ABSTRACT

In this short letter, new exact solutions including kink solutions, soliton-like solutions and periodic form solutions for a combined version of the potential KdV equation and the Schwarzian KdV equation are obtained using the generalized Riccati equation mapping method.

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1. Introduction

In recent years, nonlinear evolution equations played a major role in various fields, such as fluid mechanics, plasma physics, optical fibers, solid state physics, chemical kinematic, chemical physics and geochemistry. Concepts like solitons, kinks, breathers, cusps and compactons are now being thoroughly investigated in the scientific literature. During the past decades, many powerful methods to construct exact solutions of nonlinear evolution equations have been established and developed such as the inverse scattering transform [1], the Hirota' s bilinear operators [2], the tanh-function expansion and the Jacobi elliptic function expansion [3,4], the homogeneous balance method [5], the auxiliary function method [6,7], the exp-function expansion method [8] and so on.

In this letter, by using the generalized Riccati equation mapping method [9], we obtain some new exact solutions including kink solutions, soliton-like solutions and periodic form solutions for a combined version of the potential KdV equation and the Schwarzian KdV equation.

2. Abundant solutions by using the Riccati equation mapping method

In this section, we introduce a generalized Riccati equation mapping method to construct the exact solutions of a combined version of the potential KdV equation and the Schwarzian KdV equation [10].

The combined version of the potential KdV equation and the Schwarzian KdV equation is given as the follows:

$$u_t + \frac{9}{2}(u_x)^2 + \frac{3}{2}u_{xxx} + 2u_xS(u) = 0, \tag{2.1}$$

where $S(u)$ denotes the Schwarzian derivative of u, i.e.,

$$S(u) = \frac{u_{xxx}}{u_x} - \frac{3}{2}\frac{u_{xx}^2}{u_x^2}. \tag{2.2}$$

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We now consider Eq. (2.1) by applying the generalized Riccati equation mapping method. By using the wave variable $\xi = k(x + \omega t)$, (2.1) is converted into an ODE for $u = u(\xi)$

$$\omega(u')^2 + \frac{9}{2}k(u')^3 + \frac{7}{2}k^2u'u''' - 3k^2(u'')^2 = 0. \tag{2.3}$$

Introduce a new independent variable ϕ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i\phi^i, \tag{2.4}$$

where a_i is constants to be determined later, ϕ expresses the solution of the following generalized Raccati equation:

$$\phi'(\xi) = r + p\phi(\xi) + q\phi^2(\xi), \tag{2.5}$$

r, p and q are all real constants that would be determined by the boundary-initial value conditions for a specific problem. By balancing the highest-order linear term with the nonlinear term in (2.3), we have $m = 1$. Then, we write (2.4) in the form

$$u(\xi) = a_0 + a_1\phi, \quad a_1 \neq 0. \tag{2.6}$$

Substituting (2.6) along with (2.5) into (2.3) yields a equation for ϕ, setting the coefficients of each power of ϕ^i to zero yields

$$\frac{1}{2}k^2p^2r^2a_1^2 + 7k^2qr^3a_1^2 + r^2\omega a_1^2 + \frac{9}{2}kr^3a_1^3 = 0,$$

$$k^2p^3ra_1^2 + 23k^2pqr^2a_1^2 + 2pr\omega a_1^2 + \frac{27}{2}kpr^2a_1^3 = 0,$$

$$\frac{1}{2}k^2p^4a_1^2 + 26k^2p^2qra_1^2 + 23k^2q^2r^2a_1^2 + p^2\omega a_1^2 + 2qr\omega a_1^2 + \frac{27}{2}kp^2ra_1^3 + \frac{27}{2}kqr^2a_1^3 = 0,$$

$$10k^2p^3qa_1^2 + 50k^2pq^2ra_1^2 + 2pq\omega a_1^2 + \frac{9}{2}kp^3a_1^3 + 27kpqra_1^3 = 0,$$

$$\frac{55}{2}k^2p^2q^2a_1^2 + 25k^2q^3ra_1^2 + q^2\omega a_1^2 + \frac{27}{2}kp^2qa_1^3 + \frac{27}{2}kq^2ra_1^3 = 0,$$

$$27k^2pq^3a_1^2 + \frac{27}{2}kpq^2a_1^3 = 0,$$

$$9k^2q^4a_1^2 + \frac{9}{2}kq^3a_1^3 = 0.$$

Solving these algebraic equations, we obtain

$$a_1 = -2kq, \quad \omega = -\frac{1}{2}k^2(p^2 - 4qr). \tag{2.7}$$

Based on the solutions of (2.5), selecting different values of r, p and q we obtain abundant new types solutions for (2.1).

Type (I): Taking p, q and r satisfies $pq \neq 0$ (or $qr \neq 0$) and $p^2 - 4qr > 0,\ a_0 = -kp$, the kink solutions and the soliton-like solutions of (2.1) are recovered.

(i) The kink solutions:

$$u_1 = k\sqrt{p^2 - 4qr}\tanh\left[k\frac{\sqrt{p^2 - 4qr}}{2}\left[x - \frac{1}{2}k^2(p^2 - 4qr)t\right]\right],$$

$$u_2 = k\sqrt{p^2 - 4qr}\coth\left[k\frac{\sqrt{p^2 - 4qr}}{2}\left[x - \frac{1}{2}k^2(p^2 - 4qr)t\right]\right],$$

$$u_{3,4} = \frac{k\sqrt{p^2 - 4qr}}{2}\left(\tanh\left[\frac{k\sqrt{p^2 - 4qr}}{4}\left[x - \frac{1}{2}k^2(p^2 - 4qr)t\right]\right] \pm \coth\left[\frac{k\sqrt{p^2 - 4qr}}{4}\left[x - \frac{1}{2}k^2(p^2 - 4qr)t\right]\right]\right).$$

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