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Logic optimality for multi-objective optimization

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ABSTRACT

Pareto dominance is one of the most basic concepts in multi-objective optimization. However, it is inefficient when the number of objectives is large because in this case it leads to an unmanageable number of Pareto solutions. In order to solve this problem, a new concept of logic dominance is defined by considering the number of improved objectives and the quantity of improvement simultaneously, where probabilistic logic is applied to measure the quantity of improvement. Based on logic dominance, the corresponding logic nondominated solution is defined as a feasible solution which is not dominated by other ones based on this new relationship, and it is proved that each logic nondominated solution is also a Pareto solution. Essentially, logic dominance is an extension of Pareto dominance. Since there are already several extensions for Pareto dominance, some comparisons are given in terms of numerical examples, which indicates that logic dominance is more efficient. As an application of logic dominance, a house choice problem with five objectives is considered.

1. Introduction

The term of Pareto dominance and the related terms of Pareto solution and Pareto set are the most basic concepts in multi-objective optimization theory [2,7,8,10,12,20,23] and algorithms including both classical algorithms [15] and evolutionary algorithms [1,4,9,11,22,24,25,28]. However, when the number of objectives is large, researchers [10,20] pointed out that Pareto dominance is inefficient because it leads to an unmanageable number of Pareto solutions. The reason for the limitation is that the condition for Pareto dominance is so strict that almost no feasible solution is dominated when the number of objectives is large, which implies that almost all the solutions are Pareto solutions. In order to solve this problem, many researchers redefined the concept of dominance in a looser way by considering the number of improved objectives and the quantity of improvement.

The number of improved objectives was first considered by Parmee et al. [18], where the authors also consider an additional weighting procedure for the incorporation of preferences as crisp weighting coefficients. In 2001, Drechsler et al. [7] defined a favor dominance relationship by comparing the number of improved objectives, and defined a favor nondominated solution as a nondominated feasible solution under this new relationship. Recently, the concept of favor dominance was extended to ε-favor dominance [23], and the concept of favor nondominated solution was also extended to ε-favor nondominated solution. In 2007, Pierro et al. [20] defined a concept of efficiency of order k, that is, a feasible solution is called efficient of order k if there is no other feasible solution which improves more than k objectives.

The earliest consideration about the quantity of improvement is the well-known compromise model, which essentially transforms each multi-objective problem into a single-objective problem by a preference function. In 2004, Farina and

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Amato [10] defined a fuzzy dominance relationship in terms of measuring the quantity of improvement by a membership function. This concept was then accepted and studied widely by many researchers [5,21,28]. In addition, Yager [26] defined a dominance relationship by describing each feasible solution via a predicate formula in the Lukasiewicz logic system, and defining its fitness as the formula's truth value. This definition was also discussed and applied by many researchers [6,13]. Essentially, Yager only considered the quantity of improvement and measured it by the Lukasiewicz truth value.

The purpose of this paper is to define a new dominance relationship and a corresponding new optimal solution by using the number of improved objectives and the quantity of improvement simultaneously, where probabilistic logic is applied to measure the degree of improvement. For this purpose, this paper is organized as follows. In Section 2, some basic knowledge about multi-objective optimization, including the Pareto dominance and its extensions, are reviewed for facilitating the understanding of the paper. In addition, a brief introduction about probabilistic logic is also given. In Section 3, a concept of logic dominance is defined by measuring the quantity of improvement as probabilistic truth value. Based on this concept, a nondominated solution under this new relationship is defined, and it is proved that each of these solutions is also a Pareto solution. In order to make comparisons with Pareto dominance and their extensions, some numerical examples are presented in Section 4, which shows that logic dominance is more efficient. In Section 5, logic dominance is applied to a house choice problem.

2. Preliminaries

In this section, we introduce some basic knowledge about multi-objective optimization and probabilistic logic.

2.1. Multi-objective optimization

In multi-objective optimization problems, we attempt to optimize multiple objective functions $f_1, f_2, \ldots, f_m$ simultaneously. Without loss of generality, we assume that all the objective functions are to be maximized because minimizing f_i is equivalent to maximizing $-f_i$. In this sense, a multi-objective optimization problem is defined as

$$\begin{cases} \max[f_1(x), f_2(x), \ldots, f_m(x)] \\ \text{subject to}: \\ \qquad g_i(x) \leqslant 0, \quad i = 1, 2, \ldots, n, \end{cases} \tag{2.1}$$

where $g_i(x) \leqslant 0$ are system constraints, $i = 1, 2, \ldots, n$. In model (2.1), we call x a decision vector. The set

$$\Omega = \{x : g_i(x) \leqslant 0, i = 1, 2, \ldots, n\}$$

is called the feasible set, and each element x of Ω is called a feasible solution.

Definition 2.1. For any $x, y \in \Omega$, x is said to Pareto dominate y if and only if

(a) $f_i(x) \geqslant f_i(y)$ for all $i \in \{1, 2, \ldots, m\}$;
(b) $f_j(x) > f_j(y)$ for at least one $j \in \{1, 2, \ldots, m\}$.

Definition 2.2. A feasible solution x is said to be a Pareto solution if there is no feasible solution y such that y Pareto dominates x. The set of all Pareto solutions is called the Pareto set.

Please note that the Pareto dominance is inefficient when the number of objectives is large. We will illustrate this fact by the following examples.

Example 2.1. Assume that there is a maximization problem with 100 objectives, and there is no preference among them. Let x and y be two feasible solutions with

(a) $f_1(x) - f_1(y) = -1$;
(b) $f_i(x) - f_i(y) = 1$ for $2 \leqslant i \leqslant 100$.

Then it is clear that x is better than y. However, it follows from Definition 2.1 that y is not Pareto dominated by x. This example shows that the condition for Pareto dominance is too strict.

Example 2.2. Suppose that we would like to select a cheap, light and small notebook computer from the candidates $\{x_1, x_2, x_3, x_4\}$. The detailed data about each candidate are shown in Table 1. It is easy to prove that all the candidates are Pareto solutions such that it provides no preference information among the candidates. This example shows that the number of Pareto solution is unmanageable.

The reason for the limitation of Pareto definitions is that Pareto dominance is defined so strictly that almost no solution is dominated by the other. Hence, many researchers extend the concept of Pareto dominance.