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On the comonotonic-★-property for Sugeno integral

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ABSTRACT

The so-called comonotonic-★-property for Sugeno integral is introduced and investigated, where $\star : [0,\infty]^2 \to [0,\infty]$ is a binary operation. Under the assumption that $\lim_{b\to\infty} a \star b \in \{a,\infty\}$ and $\lim_{a\to\infty} a \star b \in \{b,\infty\}$, we prove that Sugeno integral possesses comonotonic-★-property if and only if ★ equals to one of the four operators: minimum, maximum, PF (the first projection) and PL (the last projection). We also discuss comonotonic-★-property when this assumption does not hold.

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1. Introduction

Classical Chebyshev type inequalities and their applications were investigated by many authors (see, e.g. [1,15,18,19,21]). Fuzzy Chebyshev type inequality was initiated by Flores-Franulič and Román-Flores [3], and followed by Ouyang et al. [6,9]. In [3] Flores-Franulič and Román-Flores proved a Chebyshev type inequality for Lebesgue measure-based Sugeno integral and for two continuous and strictly monotone functions (in the same sense). Based on the results of [8], Ouyang et al. [9] generalized the results of [3]. They proved the Chebyshev type inequality for arbitrarily fuzzy measure-based (the universal space is a subset of $\mathbb{R}$) Sugeno integral and for two monotone functions (in the same sense). Later on, Mesiar and Ouyang [6] obtained the following Chebyshev inequality:

$$(S)\int_A f \star g\, d\mu \geqslant (S)\int_A f\, d\mu \star (S)\int_A g\, d\mu, \tag{1.1}$$

where f, g are two comonotone functions and ★ is a binary operator bounded from above by minimun. Observe that comonotone functions can be defined in any space. Thus Mesiar and Ouyang proved the fuzzy Chebyshev inequality in an abstract space. It is known that

$$(S)\int_A f \star g\, d\mu \leqslant (S)\int_A f\, d\mu \star (S)\int_A g\, d\mu, \tag{1.2}$$

where f, g are two comonotone functions and ★ is a binary operator bounded from below by maximum (for a similar result, we refer to [10]). So it is of interest to determine when the equality

$$(S)\int_A f \star g\, d\mu = (S)\int_A f\, d\mu \star (S)\int_A g\, d\mu \tag{1.3}$$

holds for comonotone functions f, g.

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In the present article, we work along this line. Note that (1.1) and (1.2) can be seen as the domination property between the Sugeno integral and $\star$ operator restricted to comonotone functions, and (1.3) as the commuting property (for a deeper investigation of completely commuting aggregation operators we recommend [14], and for complete domination of aggregation functions we recommend [13]).

The paper is further organized as follows: After recalling some basic definitions and previous results in Section 2, we present our main results in the third section. Observe that the main results of this paper are obtained under the assumption of $\lim_{b\to\infty} a \star b \in \{a, \infty\}$ and $\lim_{a\to\infty} a \star b \in \{b, \infty\}$. So in Section 4 we discuss the case when this assumption does not hold. We close this paper by some conclusions.

2. Preliminaries

To make this work self-contained, we briefly mention some of the basic concepts and previous results.

Let X be a nonempty set and $\mathscr{F}$ be a σ-algebra of subsets of X. Let N denote the set of all positive integers and $\overline{R_+}$ denote $[0, +\infty]$. Throughout this paper, all considered subsets are supposed to belong to $\mathscr{F}$.

Fuzzy measures and fuzzy integrals were first introduced by Sugeno [17]. Since then, they were intensively investigated by many authors (see, e.g. [2,4,5,7,16]). In the following, we only recall some basic concepts. For more information on this topic, we refer to [2,11,20].

Definition 2.1. [12]A set function $\mu : \mathscr{F} \to \overline{R_+}$ is called a fuzzy measure if the following properties are satisfied:

(FM1) $\mu(\emptyset) = 0$;
(FM2) $A \subset B$ implies $\mu(A) \leqslant \mu(B)$;
(FM3) $A_1 \subset A_2 \subset \cdots$ implies $\mu(\bigcup_{n=1}^{\infty} A_n) = \lim_{n\to\infty} \mu(A_n)$; and
(FM4) $A_1 \supset A_2 \supset \cdots$, and $\mu(A_1) < +\infty$ imply $\mu(\bigcap_{n=1}^{\infty} A_n) = \lim_{n\to\infty} \mu(A_n)$.

When μ is a fuzzy measure, the triple $(X, \mathscr{F}, \mu)$ is called a fuzzy measure space.

Let $(X, \mathscr{F}, \mu)$ be a fuzzy measure space, by $\mathscr{F}_+^{\mu}(X)$ we denote the set of all nonnegative μ-measurable functions with respect to $\mathscr{F}$. In what follows, all considered functions belong to $\mathscr{F}_+^{\mu}(X)$. Let f be a nonnegative real-valued function defined on X, we will denote the set $\{x \in X | f(x) \geqslant \alpha\}$ by F_α for $\alpha \geqslant 0$. Clearly, F_α is nonincreasing with respect to α, i.e., $\alpha \leqslant \beta$ implies $F_\alpha \supset F_\beta$.

Definition 2.2. [11,17,20] Let $(X, \mathscr{F}, \mu)$ be a fuzzy measure space, the Sugeno integral of f on A, with respect to the fuzzy measure μ, is defined by

$$(S)\int_A f\, d\mu = \bigvee_{\alpha \geqslant 0} (\alpha \wedge \mu(A \cap F_\alpha)).$$

When $A = X$, then

$$(S)\int_X f\, d\mu = (S)\int f\, d\mu = \bigvee_{\alpha \geqslant 0} (\alpha \wedge \mu(F_\alpha)).$$

Some basic properties of Sugeno integral are summarized in [11,20], we cite some of them as follows.

Theorem 2.3 (*[11,20]*)

(i) *$\mu(A \cap F_\alpha) \geqslant \alpha \Rightarrow (S)\int_A f\, d\mu \geqslant \alpha$;*
(ii) *$\mu(A \cap F_\alpha) \leqslant \alpha \Rightarrow (S)\int_A f\, d\mu \leqslant \alpha$;*
(iii) *$(S)\int_A f\, d\mu < \alpha \iff$ there exists $\gamma < \alpha$ such that $\mu(A \cap F_\gamma) < \alpha$;*
(iv) *$(S)\int_A f\, d\mu > \alpha \iff$ there exists $\gamma > \alpha$ such that $\mu(A \cap F_\gamma) > \alpha$;*
(v) *If $\mu(A) < \infty$, then $\mu(A \cap F_\alpha) \geqslant \alpha \iff (S)\int_A f\, d\mu \geqslant \alpha$.*

In [9], Ouyang et al. proved the following

Theorem 2.4. *Let μ be an arbitrary fuzzy measure on [0, a] and $f, g : [0, a] \to \mathbb{R}$ be two real-valued measurable functions such that $(S)\int_0^a f\, d\mu \leqslant 1$ and $(S)\int_0^a g\, d\mu \leqslant 1$. If f, g are both nondecreasing (or both nonincreasing), then the inequality*

$$(S)\int_0^a fg\, d\mu \geqslant \left((S)\int_0^a f\, d\mu\right)\left((S)\int_0^a g\, d\mu\right) \tag{2.1}$$

holds.

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