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Observer-based finite-time control of time-delayed jump systems

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ABSTRACT

This paper provides the observer-based finite-time control problem of time-delayed Markov jump systems that possess randomly jumping parameters. The transition of the jumping parameters is governed by a finite-state Markov process. The observer-based finite-time H_∞ controller via state feedback is proposed to guarantee the stochastic finite-time boundedness and stochastic finite-time stabilization of the resulting closed-loop system for all admissible disturbances and unknown time-delays. Based on stochastic finite-time stability analysis, sufficient conditions that ensure stochastic robust control performance of time-delay jump systems are derived. The control criterion is formulated in the form of linear matrix inequalities and the designed finite-time stabilization controller is described as an optimization one. The presented results are extended to time-varying delayed MJSs. Simulation results illustrate the effectiveness of the developed approaches.

1. Introduction

A lot of dynamical systems are highly relevant to processes whose parameters are subject to random abrupt changes due to, for example, subsystem switching, system noises, sudden environment changes, failures occurred in components or interconnections, etc. Markov jump systems (MJSs) are special class of hybrid systems with two components which are the mode and the state, may be employed to model the above system phenomenon. In MJSs, the dynamics of the jump modes and continuous states are, respectively, modeled by finite-state Markov chains [9,28] and differential equations. Since the celebrated work of Krasovskii and Lidskii on quadratic control [18] in the early 1960s, MJSs regains increasing interest and there has been a dramatic progress in MJSs control theory. In practice, the applications of MJSs are comprehensive, for instance, economic systems [7], communication systems [4], electrical power systems [5], robot manipulator system [22] and circuit systems [14], etc. In the past decades, the characterization of stochastic Lyapunov stability and control issues of MJSs has been widely investigated, and the existing results cover a large variety of problems such as stochastic Lyapunov stability [12,16,21–23], stochastic controllability [5,18,24,25], robust filtering [13,15,25], etc. It is worth noticing that Rami and Ghaoui [23] started a new and prolific trend in the area of using linear matrix inequalities (LMIs) techniques [6]. But to the best of our knowledge, the stochastic finite-time stability and control problems for stochastic MJSs have not been intensively studied.

It has been recognized that time-delays are inherent features of many various practical systems, such as chemical engineering process, pneumatic systems with long transmission lines, neural network, inferred grinding model, etc. The existence of time-delays frequently causes instability in dynamic systems and usually leads to unsatisfactory performances. Therefore, the problems of stability analysis and designing controllers for time-delay systems have been of considerable

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interest and in particular robust Lyapunov stability problem of linear systems has received much consideration. As we all know, Lyapunov stability is used to deal with the asymptotic pattern of system trajectories, thus the steady-state behaviors of control systems over an infinite-time interval are paid more attention to. However, the main attention in many practical applications is the behavior of the dynamical systems over a fixed finite-time interval [10,11], for example, large values of the state are not acceptable in the presence of saturations. Therefore, we need to check the unacceptable values that the system state does not exceed a certain threshold during a fixed finite-time interval by giving some initial conditions. The concept of finite-time stability referring to these transient performances of control dynamics dates back to the Sixties, when it was introduced in the control literature [8]. Then, some attempts on finite-time stability can be found in [26] by using Lyapunov functional approach. Recently, with the aid of LMIs techniques, more concepts of finite-time stability have been proposed for linear continuous-time or discrete-time control system, such as finite-time boundedness (FTB) [3,20,27], finite-time stabilization via feedback control [2,16,20,27] and finite-time control [1,3,13,16,20,27]. In above-mentioned papers, the controlled dynamics considered are common linear systems, then the controllers are designed under the case that the total states of systems can be obtained. In actually fact, the states of system are not usually measurable in many real-world systems and these motivate us to research observer-based stochastic finite-time controller design problem of this topic.

In this paper, we discuss the observer-based stochastic finite-time analysis and synthesis problems of time-delay MJSs with norm bounded external disturbance. The observer-based finite-time H_∞ controller via state feedback is provided to guarantee the stochastic finite-time boundedness and stochastic finite-time stabilization of the resulting closed-loop system for all admissible unknown time-delays. By selecting appropriate Lyapunov–Krasovskii functions, it gives the sufficient conditions of the control criterion which can be tackled in the form of LMIs. And the presented results are then extended to time-varying delayed MJSs. At last, two numerical examples are provided to illustrate the proposed results.

In the sequel, the following notation will be used: The symbols R^n and $R^{n\times m}$ stand for an n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively, $\boldsymbol{A}^T$ and $\boldsymbol{A}^{-1}$ denote the matrix transpose and matrix inverse, diag$\{\boldsymbol{A}\quad \boldsymbol{B}\}$ represents the block-diagonal matrix of $\boldsymbol{A}$ and $\boldsymbol{B}$, $\|*\|$ denotes the Euclidean norm of vectors, $E\{\bullet\}$ denotes the mathematics statistical expectation of the stochastic process or vector, $L_2^n[0,+\infty)$ is the space of n-dimensional square integrable function vector over $[0,+\infty)$, $\boldsymbol{P}>0$ stands for a positive-definite matrix, $\boldsymbol{I}$ is the unit matrix with appropriate dimensions, 0 is the zero matrix with appropriate dimensions, $*$ means the symmetric terms in a symmetric matrix.

2. Problem formulation

Given a probability space $(\boldsymbol{\Omega},F,\boldsymbol{P})$ where $\boldsymbol{\Omega}$ is the sample space, F is the algebra of events and $\boldsymbol{P}$ is the probability measure defined on F. Let the random form process $\{\boldsymbol{r}_t, t\geqslant 0\}$ be a continuous-time discrete-state homogeneous Markov stochastic process taking values on a finite set $\boldsymbol{M}=\{1,2,\ldots,N\}$ with transition rate matrix $\boldsymbol{\Pi}=\{\pi_{ij}\}$, $i,j\in\boldsymbol{M}$ and having the additional property $P_r(\{\boldsymbol{r}_0=i\})>0$, $\forall i\in\boldsymbol{M}$. Thus, we can define the following transition probability from mode i at time t to mode j at time $t+\Delta t$ as

$$\boldsymbol{P}_{ij}=P_r\{\boldsymbol{r}_{t+\Delta t}=j|\boldsymbol{r}_t=i\}=\begin{cases}\pi_{ij}\Delta t+\boldsymbol{o}(\Delta t), & i\neq j,\\ 1+\pi_{ii}\Delta t+\boldsymbol{o}(\Delta t), & i=j,\end{cases}\tag{1}$$

with transition probability rates $\pi_{ij}\geqslant 0$ for $i,j\in\boldsymbol{M}$, $i\neq j$ and $\sum_{j=1,j\neq i}^{N}\pi_{ij}=-\pi_{ii}$ where $\Delta t>0$ and $\lim_{\Delta t\downarrow 0}\boldsymbol{o}(\Delta t)/\Delta t\to 0$.

Consider the following time-delay MJSs in the probability space $(\boldsymbol{\Omega},F,\boldsymbol{P})$:

$$\begin{cases}\dot{\boldsymbol{x}}(t)=\boldsymbol{A}(\boldsymbol{r}_t)\boldsymbol{x}(t)+\boldsymbol{A}_d(\boldsymbol{r}_t)\boldsymbol{x}(t-\tau(t))+\boldsymbol{B}(\boldsymbol{r}_t)\boldsymbol{u}(t)+\boldsymbol{B}_d(\boldsymbol{r}_t)\boldsymbol{d}(t),\\ \boldsymbol{z}(t)=\boldsymbol{C}(\boldsymbol{r}_t)\boldsymbol{x}(t)+\boldsymbol{C}_d(\boldsymbol{r}_t)\boldsymbol{x}(t-\tau(t))+\boldsymbol{D}(\boldsymbol{r}_t)\boldsymbol{u}(t)+\boldsymbol{D}_d(\boldsymbol{r}_t)\boldsymbol{d}(t),\\ \boldsymbol{y}(t)=\boldsymbol{C}_y(\boldsymbol{r}_t)\boldsymbol{x}(t)+\boldsymbol{C}_{yd}(\boldsymbol{r}_t)\boldsymbol{x}(t-\tau(t)),\\ \boldsymbol{x}(t)=\boldsymbol{\eta}(t), t\in[-\bar{\tau}\quad 0], \boldsymbol{r}_t=\boldsymbol{r}_0,\quad t=0,\end{cases}\tag{2}$$

where $\boldsymbol{x}(t)\in R^n$ is the state, $\boldsymbol{z}(t)\in R^l$ is the controlled output, $\boldsymbol{y}(t)\in R^q$ is the measured output, $\boldsymbol{u}(t)\in R^m$ is the controlled input, $\boldsymbol{d}(t)\in L_2^p[0\quad +\infty)$ is the external disturbances, $\boldsymbol{\eta}(t)\in L_2^n[-\tau\quad 0]$ is a continuous vector-valued initial function, $\boldsymbol{r}_0$ is the initial mode, $\boldsymbol{A}(\boldsymbol{r}_t)$, $\boldsymbol{A}_d(\boldsymbol{r}_t)$, $\boldsymbol{B}(\boldsymbol{r}_t)$, $\boldsymbol{B}_d(\boldsymbol{r}_t)$, $\boldsymbol{C}(\boldsymbol{r}_t)$, $\boldsymbol{C}_d(\boldsymbol{r}_t)$, $\boldsymbol{D}(\boldsymbol{r}_t)$, $\boldsymbol{D}_d(\boldsymbol{r}_t)$, $\boldsymbol{C}_y(\boldsymbol{r}_t)$, $\boldsymbol{C}_{yd}(\boldsymbol{r}_t)$ are known mode-dependent constant matrices with appropriate dimensions. $\tau(t)$ is a kind of positive time-varying differentiable bounded delays [17,19] which can be described as

$$0<\tau(t)\leqslant\bar{\tau}<\infty,\quad \dot{\tau}(t)\leqslant 1.\tag{3}$$

For convenience, we denote $\boldsymbol{A}(\boldsymbol{r}_t)$, $\boldsymbol{A}_d(\boldsymbol{r}_t)$, $\boldsymbol{B}(\boldsymbol{r}_t)$, $\boldsymbol{B}_d(\boldsymbol{r}_t)$, $\boldsymbol{C}(\boldsymbol{r}_t)$, $\boldsymbol{C}_d(\boldsymbol{r}_t)$, $\boldsymbol{D}(\boldsymbol{r}_t)$, $\boldsymbol{D}_d(\boldsymbol{r}_t)$, $\boldsymbol{C}_y(\boldsymbol{r}_t)$, $\boldsymbol{C}_{yd}(\boldsymbol{r}_t)$ as $\boldsymbol{A}_i$, $\boldsymbol{A}_{di}$, $\boldsymbol{B}_i$, $\boldsymbol{B}_{di}$, $\boldsymbol{C}_i$, $\boldsymbol{C}_{di}$, $\boldsymbol{D}_i$, $\boldsymbol{D}_{di}$, $\boldsymbol{C}_{yi}$, $\boldsymbol{C}_{ydi}$.

Assumption 2.1. The external disturbance $d(t)$ is varying and satisfies the following constraint condition:

$$\int_0^T \boldsymbol{d}^T(t)\boldsymbol{d}(t)\,dt\leqslant d,\quad d\geqslant 0.\tag{4}$$