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The dynamics of an impulsive one-prey multi-predators system with delay and Holling-type II functional response

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ABSTRACT

Considering the effects of changing environments, delays, impulses and functional response, a one-prey multi-predators system is established in this paper. Using comparison theorem and some analysis techniques, sufficient conditions ensuring the global attractivity of the prey-extinction positive periodic solution and the permanence of the system are obtained. Finally, examples and numerical simulations are given to show the effectiveness of the main results.

1. Introduction

In the last decades, the predator–prey model has been studied extensively after the pioneering work of Volterra and Lotka for predator–prey interactions in the middle of the 20th century [1–3]. Since the ecological system is often deeply perturbed by activities of human exploitation (planting and harvesting), one needs to use impulsive differential equations to accurately describe the system. Impulsive methods have been applied in almost every field of applied sciences, see [4,5] and references therein. Recently, impulsive differential equations have been intensively researched and many nice results are obtained, see [6–9,11,12,14,15,20,22,23].

On the other hand, in population dynamics, a functional response of the predator to the prey density refers to the change in the density of prey attached per unit time per predator as the prey density changes. Functional response often affects the dynamics of biological system and different functional responses have been studied by many scholars, such as Holling-type [8,9,15], Watt-type [10–12] and Ivlev-type [13,14]. Currently, the field of research on the dynamics of impulsive differential equations with functional responses seems to be a new increasingly interesting area, see [7–13,23].

However, further scientific research suggests that time delay differential equations exhibit much more complicated dynamics than ordinary differential equations [16–19,23]. For example, in [23], considering the effects of changing environment and delays, the authors discussed the following general two-species nonautonomous predator–prey model with multi-delays and impulses:

$$\begin{cases} \left.\begin{aligned} x'(t) &= x(t)\left(r_1(t) - \sum_{i=1}^{n_1} a_{1i}(t)x(t-\tau_{1i})\right) - \frac{\beta(t)x(t)y(t)}{1+\alpha(t)x(t)} \\ y'(t) &= y(t)\left(r_2(t) + \frac{\beta(t)x(t-\tau)}{1+\alpha(t)x(t-\tau)} - \sum_{i=1}^{n_2} a_{2i}(t)y(t-\tau_{2i})\right) \end{aligned}\right\}, & t \neq t_k, \\ \left.\begin{aligned} x(t_k^+) - x(t_k) &= b_{1k}x(t_k) \\ y(t_k^+) - y(t_k) &= b_{2k}y(t_k) \end{aligned}\right\}, & t = t_k,\ k = 1,2,\ldots \end{cases}$$

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But in the real word, multi-predators often coexist in an ecological system, see [14,15,17–19] and references cited therein. For example, Wu and Huang [14] studied the dynamics of one-prey multi-predators impulsive system with Ivlev-type functional response; Pei et al. [15] studied the extinction and permanence of one-prey multi-predators system with Holling-type II functional response. Therefore, it is more realistic and interesting for us to consider one-prey multi-predators model with effects of the seasonality of changing environments, impulses, time delays and functional response.

Based on the above discussion, in this paper, we are concerned with the following impulsive one-prey multi-predators system with delays and Holling-type II functional response:

$$\begin{cases} \left.\begin{array}{l} x'(t) = x(t)(r_0(t) - b_0(t)x(t-\tau_0)) - \sum\limits_{i=1}^{n} \frac{x(t)y_i(t)}{a_i(t)+x(t)} \\ y_i'(t) = y_i(t)\left(r_i(t) - b_i(t)y_i(t-\tau_i) + \frac{\alpha_i(t)x\left(t-\tau_i^*\right)}{a_i(t)+x\left(t-\tau_i^*\right)}\right) \end{array}\right\}, \quad t \neq t_k \\ \left.\begin{array}{l} x(t_k^+) = (1+q_{0k})x(t_k) \\ y_i(t_k^+) = (1+q_{ik})y(t_k) \end{array}\right\}, \quad t = t_k,\ i = 1,2,\ldots,n,\ k = 1,2,\ldots \end{cases} \tag{1.1}$$

with initial conditions

$$(x(s), y_1(s), y_2(s), \ldots, y_n(s)) = (\phi_0(s), \phi_1(s), \phi_2(s), \ldots, \phi_n(s)) \in C\left([-\tau, 0], R_+^{n+1}\right), \quad \phi_0(0) > 0,\ \phi_i(0) > 0,$$

where $x(t)$, $y_i(t)$ represent the densities of prey and the ith predator, respectively. $r_0(t)$, $b_0(t)$, $b_i(t)$, $a_i(t)$ and $\alpha_i(t)$ are all positive, continuous and ω-periodic functions, $a_i(t)$ is the half-saturation function for the ith predators. $r_i(t)$ is continuous and ω-periodic function, $r_i(t) > 0$ represents the logistic intrinsic growth rate of the ith predators in the absence of the prey, $r_i(t) < 0$ represents the death rate of the ith predators. $\alpha_i(t)$ is the conversion coefficient of the amount of consumed prey transferred to the ith newborn predators. $\tau_0, \tau_i, \tau_i^* > 0$ are constants, $\tau = \max\{\tau_0, \tau_i, \tau_i^*,\ 1 \leqslant i \leqslant n\}$, $R_+^{n+1} = \{x \in R^{n+1} : x \geqslant 0\}$, $i = 1, 2, \ldots, n$. For more details on the ecological sense of system (1.1), we refer to [7–9,15,18,23].

It is easily verified that solutions of system (1.1) are positive for all $t \geqslant 0$. The main purpose of this paper is to study the global attractivity of prey-extinction periodic solution and the permanence of system (1.1). Now we give some definitions.

Definition 1.1. Functions $x, y_i \in C([-\tau, \infty), [0, \infty))$ are said to be a solution of system (1.1) on $[-\tau, \infty)$ provided:

(i) $x(t)$ and $y_i(t)$ are absolutely continuous on each interval $(0, t_1]$ and $(t_k, t_{k+1}]$.
(ii) For any t_k, $x(t_k^+)$, $y_i(t_k^+)$, $x(t_k^-)$ and $y_i(_k^-)$ exist and satisfying $x(t_k^-) = x(t_k), y_i(t_k^-) = y_i(t_k)$ for $i = 1,2,\ldots,n$, $k = 1,2,\ldots$
(iii) $x(t)$, $y_i(t)$ satisfy (1.1) almost everywhere in $[0, \infty)/\{t_k\}$ and satisfy $x(t_k^+) = (1+q_{0k})x(t_k)$, $y_i\left(t_k^+\right) = (1+q_{ik})y_i(t_k)$ for $i = 1,2,\ldots,n$, $k = 1,2,\ldots$

Definition 1.2. System (1.1) is said to be permanent if there exist positive constants ξ and η with $\eta > \xi > 0$ such that each positive solution $(x(t), y_i(t))$ satisfying $\xi \leqslant x(t) \leqslant \eta$, $\xi \leqslant y_i(t) \leqslant \eta$ for t sufficiently large and $i = 1,2,\ldots,n$.

Throughout this paper, we assume that:

(H_1) $0 < t_1 < t_2 < \cdots < t_n < \cdots$ are fixed impulsive points with $\lim_{t\to\infty} t_k = \infty$.
(H_2) q_{0k}, q_{ik} are real sequences with $q_{0k} > -1$, $q_{ik} > -1$ for $i = 1,2,\ldots,n$, $k = 1,2,\ldots$
(H_3) $\Pi_{0<t_k<t}(1+q_{0k})$ and $\Pi_{0<t_k<t}(1+q_{ik})$ are ω-periodic functions and satisfy

$$m_0 \leqslant \Pi_{0<t_k<t}(1+q_{0k}) \leqslant M_0, \quad m_i \leqslant \Pi_{0<t_k<t}(1+q_{ik}) \leqslant M_i,$$

where m_0, m_i and M_0, M_i are positive constants, $i = 1,2,\ldots,n$, $k = 1,2,\ldots$

Here and in the sequel, we suppose that a product equals unit if the number of factors is equal to zero.

For convenience, we cite the following notations.

$$\delta_0 = \frac{r_0^m}{b_0^M M_0}, \quad \delta_i = \frac{r_i^m}{b_i^M M_i}, \quad \Delta_0 = \frac{r_0^M}{b_0^m m_0}, \quad \Delta_i = \frac{r_i^M}{b_i^m m_i},$$

$$H_i = M_i \Delta_i e^{r_i^M \tau_i}, \quad \eta_0 = \Delta_0 e^{r_0^M \tau_0} M_0, \quad G_i = \frac{\alpha_i^M \eta_0}{a_i^m + \eta_0}, \quad N_i = \frac{\alpha_i^m \xi_0}{a_i^m + \xi_0},$$

$$\eta_i = \left(\Delta_i + \frac{G_i}{b_i^m m_i}\right) e^{(r_i^M + G_i)\tau_i}, \quad \beta_i = m_i \delta_i e^{(r_i^m - H_i r_i^M)\tau_i},$$

$$\xi_0 = \left(\delta_0 - \sum_{i=1}^{n} \frac{\eta_i}{b_0^M M_0 a_i^M}\right) e^{\left(r_0^m - \sum\limits_{i=1}^{n} \frac{\eta_i}{a_i^M} - \eta_0 b_0^M\right)\tau_0},$$

$$\xi_i = \left(\delta_i + \frac{N_i}{b_i^M M_i}\right) e^{(r_i^m + N_i - \eta_i b_i^M)\tau_i}, \quad f^M = \max_{0\leqslant t\leqslant\omega} f(t), \quad f^m = \min_{0\leqslant t\leqslant\omega} f(t),$$