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Module-phase synchronization in complex dynamic system

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A R T I C L E I N F O

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A B S T R A C T

The concept of module-phase synchronization was proposed. The chaos synchronization between drive system and response system was achieved in module space and phase space respectively (module-phase synchronization). Different from the evolutions in real space, there is no pseudorandom behavior in phase space when module-phase synchronization achieve. All the phases of complex state variables switched between two fixed values which are determined by initial values of drive system. And the modules varied within a bounded field. The theoretical analysis and the simulations were also given.

1. Introduction

Chaos is a ubiquitous phenomenon in nonlinear system. Since 1990, the concept of chaotic synchronization was proposed by Pecora and Carroll [1], the chaotic synchronization has been investigated intensively [2–11]. It is known that synchronization play a very important role in many different contexts such as biology, ecology, climatology, sociology, technology, or even in arts [5]. Many synchronization methods have been proposed, such as general synchronization [12]; complete synchronization [13]; phase synchronization [14–16], high precision fast synchronization [17]; lag synchronization [18] and projective synchronization etc. [19–25]. However, much attention has focused on the projective synchronization in real.

Recently, the synchronization of complex nonlinear dynamical system has been investigated [26,27]. However, because of the complexity of the chaotic complex system, in [26] and [27], the synchronization of imaginary part and the real part were investigated respectively. In fact, this synchronization still is the synchronization of real chaotic system. In practice, sometimes, our major concerns are module and phase instead of imaginary part and real part. For example, in alternating current machine, the current amplitude is represented by module, and the current phase is described by the phase of corresponding complex number. And the observable or measurable data are also modules and phases in practice. Thus, synchronizations of module and phase are very important in complex nonlinear dynamical system.

In real space, the chaos phenomenon and its synchronization have been studied extensively. However, what the behaviors are in complex space? Can we synchronize two complex chaotic systems in complex space? As we known, in complex space, there are two important quantities - module and phase. Thus, the behaviors of module and phase should be investigated primarily. Surprisingly, it is found that the evolutions of phases and states are entirely different. The phases have no pseudorandom sequence when two complex chaotic systems achieve synchronization in module and phase. Experiments show that all the phases of complex state variables switched between two fixed values which are determined by initial values of drive system, and the module evolve on some special attractors.

In this paper, the concept of module-phase synchronization was proposed, and the module-phase synchronization of chaotic complex system coupled through single real state variable was investigated. The controllers were designed to realize module-phase synchronization when the complex dynamic system achieves chaos.

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The rest of the paper is organized as following. In Section 2, the concept of module-phase synchronization in complex dynamic systems which coupled through single real variable was proposed. The theoretical analysis and proof were also given. Then, in Section 3, numerical simulations were implemented. Finally, conclusions were given in Section 4.

2. Module-phase synchronization of complex dynamic systems coupled through single real variable

Consider two complex dynamical systems:

$$\dot{\eta} = \psi(\eta), \tag{a}$$

$$\dot{\xi} = \omega(\xi), \tag{b}$$

where, $\eta = (\eta_1, \eta_2, \ldots, \eta_m)^T \in C^m$ and $\xi = (\xi_1, \xi_2, \ldots, \xi_m)^T \in C^m$ are complex state vectors, ψ and ω are complex nonlinear function.

Definition 1. For the drive system (a) and response system (b), the two complex dynamical systems are said to achieve module-phase synchronization (MPS), if $\lim_{t\to\infty}(M(\xi_i) - M(\eta_i)) = 0$ and $\lim_{t\to\infty}(P(\xi_i) - P(\eta_i)) = 0$. Where $M(\xi_i)$ and $M(\eta_i)$ are module of ξ_i and η_i respectively, $P(\xi_i)$ and $P(\eta_i)$ are phases of ξ_i and η_i respectively.

Consider following complex dynamic systems which coupled through single variable. Drive system:

$$\begin{cases} \dot{\eta} = \psi(\eta, z), \\ z = \varphi(\eta, z), \end{cases} \tag{1a}$$

Response system:

$$\begin{cases} \dot{\xi} = \psi(\xi, z) + \boldsymbol{W}, \\ z = \varphi(\eta, z). \end{cases} \tag{1b}$$

Here, $\eta = (\eta_1, \eta_2, \ldots, \eta_m)^T \in C^m$ and $\xi = (\xi_1, \xi_2, \ldots, \xi_m)^T \in C^m$ are complex state vectors, $z \in R^1$ is real state variable, $\boldsymbol{W} = (w_1, w_2, \ldots, w_m)^T \in C^m$ is control vector, $\psi : C^m \times R^1 \to C^m$ and $\varphi : C^m \times R^1 \to R^1$ are complex nonlinear function.

As we know, in real, the goal of synchronization is achieving synchronization between corresponding variables of drive system and response system. Similarly, the goal of synchronization of complex dynamic system is achieving synchronization between the corresponding complex states of drive system and response system. Conventional method is synchronizing real part and imaginary part respectively [26,27]. In practice, sometimes, we just care the module and phase of complex variable instead of real part and imaginary part. Certainly, the synchronization of module and phase can be ensured by synchronization of real part and imaginary part. However, usually, the real part and the imaginary part are not measurable or observable. For example, in dynamos system, the measurable physical quantity is current amplitude (module) and phase instead of its real part and imaginary part. Thus, the module-phase synchronization of complex dynamic system is very important.

In real, rewrite the systems (1a) and (1b) as follows:

The drive system is:

$$\begin{cases} \dot{\boldsymbol{X}} = F(\boldsymbol{X}, t), \\ \dot{z} = H(\boldsymbol{X}, z). \end{cases} \tag{2a}$$

And the response system is:

$$\begin{cases} \dot{\boldsymbol{Y}} = F(\boldsymbol{Y}, t) + \boldsymbol{U}(\boldsymbol{X}, \boldsymbol{Y}), \\ \dot{z} = H(\boldsymbol{Y}, z). \end{cases} \tag{2b}$$

Here, $\boldsymbol{X} = (x_1, x_2, \ldots, x_{2m})^T \in R^{2m}$ and $\boldsymbol{Y} = (y_1, y_2, \ldots, y_{2m})^T \in R^{2m}$ are real state vectors, z is real state variable, $\boldsymbol{U}(\boldsymbol{X}, \boldsymbol{Y}) = (u_1, u_2, \ldots, u_{2m})^T \in R^{2m}$ is real control vector, $F : R^{2m} \times R^+ \to R^{2m}$ is a smooth nonlinear vector field, $H : R^{2m} \times R^1 \to R^1$ is a nonlinear function, $\boldsymbol{U} : R^{2m} \times R^{2m} \to R^{2m}$ is controller. Correspondingly, the *ith* complex state variables of drive system and response system are $\eta_i = x_{2i-1} + jx_{2i}, (i = 1, 2, \ldots, m)$ and $\xi_i = y_{2i-1} + jy_{2i}, (i = 1, 2, \ldots, m)$ respectively, and the *ith* complex control vector of response system is $w_i = u_{2i-1} + ju_{2i}, (i = 1, 2, \ldots, m)$. Here, $j = \sqrt{-1}$.

Suppose $r_{\eta i} = M(x_{2i-1}, x_{2i})$ and $r_{\xi i} = M(y_{2i-1}, y_{2i})$, $(i = 1, 2, \ldots, m)$, are the module of the η_i and ξ_i respectively. $\phi_{\eta i} = \varphi(x_{2i-1}, x_{2i})$ and $\phi_{\xi i} = \varphi(y_{2i-1}, y_{2i})$, $(i = 1, 2, \ldots, m)$ are the phase of η_i and ξ_i respectively.

Here,

$$M(x, y) = \sqrt{x^2 + y^2}, \tag{3}$$

denotes the module of $x + jy$.

$$\varphi(x, y) = \begin{cases} \arctan(y/x) & (x > 0,\ y \geqslant 0) \\ 2\pi + \arctan(y/x) & (x > 0,\ y < 0) \\ \pi + \arctan(y/x) & (x < 0,\ y \neq 0) \end{cases} \tag{4}$$