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Robust exponential stability and stabilizability of linear parameter dependent systems with delays

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ABSTRACT

The robust exponential stability and stabilizability problems are addressed in this paper for a class of linear parameter dependent systems with interval time-varying and constant delays. In this paper, restrictions on the derivative of the time-varying delay is not required which allows the time-delay to be a fast time-varying function. Based on the Lyapunov–Krasovskii theory, we derive delay-dependent exponential stability and stabilizability conditions in terms of linear matrix inequalities (LMIs) which can be solved by various available algorithms. Numerical examples are given to illustrate the effectiveness of our theoretical results.

1. Introduction

Time-delay systems are frequently encountered in various areas such as chemical engineering systems, biological modeling, economics and many others [2,4,5,8–23]. It is well known that the existences of time delay in a system may cause instability and oscillations system. In the past decades, stability analysis for time-delay systems has been investigated extensively and various approaches to such problems have been proposed, see [2,4,5,8,9,11–23] and the references cited therein. In particular, the exponential stability analysis of time-delay systems has been studied by many researchers in the past decades, see for examples [2,4,8,14–16,20,21,23].

In practical, there are uncertainties which have an effect on the performance of the system such as noise or external forces. The study of stability of systems with uncertainties is called robust stability. An important type of uncertainties is called parametric uncertainties in which the uncertain state matrices of systems with this type of uncertainties are in the polytope consisting of all convex combinations of known matrices.

As a result, it is important and interesting to investigate the robust stability of systems with parametric uncertainties. The linear system with polytopic-type uncertainties is called linear parameter dependent (LPD) system. There are several applications of LPD systems such as controller design for a diesel engine using VGT/EGR [1], filter design problems [6], and stability analysis of induction motor [3]. For further study on stability analysis of linear uncertain polytopic systems with delay, see [4,7,11,12,16] and references cited therein.

In this paper, we propose to study the robust exponential stability and stabilization of linear parameter dependent system with constant delay and interval time-varying delays. The restriction on the derivative of the interval time-varying delay is removed in this paper (which is often assumed to be less than one in the literatures), which allows the time-delay to be a fast time-varying function. Based on the Lyapunov–Krasovskii functional and Newton–Leibniz formula, we derive the

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delay-dependent stability criteria for the system under consideration. The derived criteria is in terms of LMI which can be solved by various algorithm such as the LMI Toolbox in Matlab. Then, we extend to obtained result to establish a robust exponential stability and stabilization criteria for a system with polytopic-type uncertainties. Finally, we provide numerical examples to demonstrate the effectiveness and applicability of theoretical results.

Notation: Throughout this paper, R^+ denotes the set of all real non-negative numbers; for real symmetric matrices X and Y, the notation $X \geqslant Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). The notation A^T represents the transpose of matrix A. We use $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ to denote the minimum and maximum eigenvalues of a real symmetric matrix, respectively. For $x \in \mathbb{R}^n$, the norm of x, denoted by $\|x\|$, is defined by $\|x\| = \left(\sum_{i=1}^n x_i^2\right)^{\frac{1}{2}}$; $\|\cdot\|_{cl} = \sup_{-h \leqslant \theta \leqslant 0}\{\|x(t+\theta)\|, \|\dot{x}(t+\theta)\|\}$.

2. Problem statement and main result

We consider the following linear parameter dependent system with delays

$$\begin{cases} \dot{x}(t) = A(\alpha)x(t) + B(\alpha)x(t-d) + C(\alpha)x(t-h(t)), & t \in R^+, \\ x(t) = \phi(t), & t \in [-\tau, 0], \end{cases} \tag{2.1}$$

where $x(t) \in R^n$ is the state vector, $A(\alpha), B(\alpha), C(\alpha) \in \mathbb{R}^{n\times n}$. The matrices $A(\alpha)$, $B(\alpha)$ and $C(\alpha)$ are subject to uncertainties and satisfy the real convex polytopic model

$$[A(\alpha) \quad B(\alpha) \quad C(\alpha)] \in \Omega, \quad \Omega = \left\{ \left[\sum_{i=1}^N \alpha_i A_i \quad \sum_{i=1}^N \alpha_i B_i \quad \sum_{i=1}^N \alpha_i C_i \right], \sum_{i=1}^N \alpha_i = 1, \quad \alpha_i \geqslant 0 \right\}, \tag{2.2}$$

where A_i, B_i and C_i $(i = 1,2,\ldots,N)$ are constant matrices with appropriate dimensions and α_i $(i = 1,2,\ldots,N)$ are time-invariant uncertainties. The delay d is positive constant and time-varying delay $h(t)$ is a time-varying continuous function that satisfies

$$0 \leqslant h_1 \leqslant h(t) \leqslant h_2, \quad 0 \leqslant d, \tag{2.3}$$

where h_1, h_2 are constants. The initial condition function $\phi(t)$ denotes a continuous vector-valued initial function of $t \in [-\tau, 0]$, where $\tau = max\{h_2, d\}$.

Defining $h_a = \frac{1}{2}(h_1 + h_2)$ and $h_r = \frac{1}{2}(h_2 - h_1)$, $h(t)$ is satisfying (2.3) can be expressed as

$$h(t) = h_a + h_r \tau(t), \tag{2.4}$$

where

$$\tau(t) = \begin{cases} \frac{2h(t) - (h_2 + h_1)}{h_2 - h_1}, & h_2 > h_1, \\ 0, & h_2 = h_1. \end{cases}$$

Obviously, $|\tau(t)| \leqslant 1$. For this case, $h(t)$ is a function belonging to the interval $[h_a - h_r, h_a + h_r]$, where h_r can be taken as the range of variation of the time-varying delay $h(t)$. Using the Newton–Leibniz formula gives

$$x(t-h(t)) = x(t-h_a) - \int_{t-h(t)}^{t-h_a} \dot{x}(s)ds, \tag{2.5}$$

from which system (2.1) can be rewritten as

$$\dot{x}(t) = A(\alpha)x(t) + B(\alpha)x(t-d) + C(\alpha)x(t-h_a) - C(\alpha)\int_{t-h(t)}^{t-h_a} \dot{x}(s)ds. \tag{2.6}$$

Note that the system (2.6) requires initial function $\psi(t)$ on $[-2\tau, 0] : \psi(s) = \phi(s + h(0)),\ -h_2 - h(0) \leqslant s \leqslant -h(0),\ \psi(s) = x(t+s),\ -h(0) \leqslant s \leqslant 0$, and as shown in Hale and Verduyn Lunel [10], it is a special case of the system (2.1) such that the stability property of the system (2.6) will ensure the stability property of the system (2.1). Therefore, we will consider the stability of the system (2.6) in order to ascertain the stability of system (2.1).

2.1. Robust exponential stability

For the stability of system (2.1), we now state and establish the following result.

Theorem 2.1. *For given non-negative scalars h_1, h_2 and d, $\beta > 0$, system (2.1) with constant matrices A, B and C and a time-varying delay satisfying (2.3) is exponentially stable with decay rate β if there exist positive definite matrices P, Q, R, U, S, W and matrices T_i, N_i and M_i, $(i = 1,2,3,4)$ with appropriate dimensions such that the following LMI holds:*