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Norms of multiplication operators on Hardy spaces and weighted composition operators from Hardy spaces to weighted-type spaces on bounded symmetric domains

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ABSTRACT

Let $\mathcal{D}$ be a bounded symmetric domain. We calculate operator norm of the multiplication operator on the Hardy space $H^p(\mathcal{D})$, as well as of the weighted composition operator from $H^p(\mathcal{D})$ to a weighted-type space.

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1. Introduction

Let $\mathcal{D}$ be a bounded symmetric domain in $\mathbb{C}^n$ in its Harish–Chandra realization and $0 \in \mathcal{D}$, $\mathbb{B} = \mathbb{B}^n$ the open unit ball in $\mathbb{C}^n$, and $\mathbb{D}^n$ the open unit polydisk in $\mathbb{C}^n$. Let $z = (z_1, \ldots, z_n)$ and $w = (w_1, \ldots, w_n)$ be points in $\mathbb{C}^n$, $\langle z, w\rangle = \sum_{k=1}^n z_k \bar{w}_k$ and $|z| = \sqrt{\langle z, z\rangle}$. Let $H(\mathcal{D})$ be the class of all holomorphic functions on $\mathcal{D}$.

Any bounded symmetric domain $\mathcal{D}$, furnished with the Bergman metric ρ, is a hermitian symmetric space $(\mathcal{D}, \rho)$ of non-compact type and is necessarily simply connected [7, p. 311]. Let Γ be the group of holomorphic automorphisms of $\mathcal{D}$. Group Γ is transitive on $\mathcal{D}$ and extends continuously to the topological boundary $\partial\mathcal{D}$ of $\mathcal{D}$ [10, p. 269]. The isotropy group $\Gamma_0 = \{\gamma \in \Gamma : \gamma(0) = 0\}$ of Γ is a compact subgroup of Γ and contains no normal subgroup of Γ. Thus $\mathcal{D}$ can be identified with the coset space Γ/Γ_0. Each bounded symmetric domain $\mathcal{D}$ has its irreducible decomposition $\mathcal{D} = \mathcal{D}_1 \times \cdots \times \mathcal{D}_m$ where each domain $\mathcal{D}_j \subset \mathbb{C}^{n_j}$, $j = 1, \ldots, m$, is irreducible. A bounded symmetric domain $\mathcal{D}$ is circular and convex and has Šilov boundary b which is circular and invariant under Γ [10]. The group Γ_0 is transitive on b [38, p. 922] and b has a unique normalized Γ_0-invariant measure σ (i.e., $\sigma(b) = 1$).

As proved in [8] $\mathcal{D}$ has a Szegö kernel, i.e., for each fixed $z \in \mathcal{D}$ there exists a function $S_z(w) = S(z, w) \in H(\mathcal{D} \times \mathcal{D}) \cap C(\mathcal{D} \times \overline{\mathcal{D}})$ such that

$$f(z) = \int_b f(\zeta)\overline{S_z(\zeta)}d\sigma(\zeta), \tag{1}$$

for all $f \in H(\mathcal{D}) \cap C(\overline{\mathcal{D}})$.

For $p > 0$ the Hardy space $H^p(\mathcal{D})$ is defined on $\mathcal{D}$ by

$$H^p(\mathcal{D}) = \left\{ f : f \in H(\mathcal{D}), \|f\|_{H^p(\mathcal{D})} := \sup_{0 \leqslant r < 1} \left(\int_b |f(r\zeta)|^p d\sigma(\zeta) \right)^{1/p} < \infty \right\}.$$

Each $f \in H^p(\mathcal{D})$ has finite limit $\lim_{r\to 1} f(r\zeta)$ a. e. on $\zeta \in b$. The boundary function we will also denote by f.

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By $H^\infty(\mathcal{D})$ we denote the space of all bounded holomorphic functions on $\mathcal{D}$ with the supremum norm

$$\|f\|_{H^\infty(\mathcal{D})} = \sup_{z\in\mathcal{D}} |f(z)|.$$

The weighted-type space $H^\infty_\mu(\mathcal{D})$ consists of all $f \in H(\mathcal{D})$ such that

$$\|f\|_{H^\infty_\mu(\mathcal{D})} = \sup_{z\in\mathcal{D}} \mu(z)|f(z)| < \infty,$$

where μ is a positive continuous function on $\mathcal{D}$ (*weight*).

Let X be a domain in $\mathbb{C}^n$, $u \in H(X)$ and φ be a holomorphic self-map of X. For any $f \in H(X)$ the weighted composition operator is defined by

$$(uC_\varphi f)(z) = u(z)f(\varphi(z)), \quad z \in X.$$

For $\varphi(z) \equiv z$ the operator is reduced to the multiplication operator $M_u f(z) = u(z)f(z)$, while for $u(z) \equiv 1$ it is reduced to the composition operator $C_\varphi f(z) = f(\varphi(z))$. A standard problem is to provide function theoretic characterizations when u and φ induce bounded or compact weighted composition operators on spaces of holomorphic functions. For some classical results in the topic, see, e.g., [5]. For some recent results in $\mathbb{C}^n$, see, e.g., [1–4,11,14,16,17,20,23,25,26,29–33,35,37,39,41] and the references therein.

One of the interesting questions is to find the exact value of operator norm of a specific linear operator such as composition operator, multiplication operator, weighted composition operator, integral-type operator etc. Majority of papers in the area only find asymptotics of the operator norm of certain linear operators on some spaces of holomorphic functions, while there are only several papers which calculate exact values of the norm of these operators, see, e.g., [2,5,13,19,20,22,24,26,29,30,32,36] (see also related results in [15,27,31,34]). For some related operators see, e.g., [18,21,27,28].

Recently in [20], motivated by our paper [11], we calculated the norm of the operator $uC_\varphi : \mathcal{B}(\mathbb{B})(\text{or } \mathcal{B}_0(\mathbb{B})) \to H^\infty_\mu(\mathbb{B})$, where $\mathcal{B}(\mathbb{B})$ is the Bloch space and $\mathcal{B}_0(\mathbb{B})$ is the little Bloch space on $\mathbb{B}$. This motivated us to start with systematic investigation of methods for calculating the norms of weighted composition, integral-type and other closely related operators between various spaces of holomorphic functions.

The following results were proven in [31].

Theorem 1. *Assume $p > 0$, $u \in H(\mathbb{B})$, μ is a weight, φ is a holomorphic self-map of $\mathbb{B}$ and $uC_\varphi : H^p(\mathbb{B}) \to H^\infty_\mu(\mathbb{B})$ is bounded. Then*

$$\|uC_\varphi\|_{H^p(\mathbb{B})\to H^\infty_\mu(\mathbb{B})} = \sup_{z\in\mathbb{B}} \frac{\mu(z)|u(z)|}{(1-|\varphi(z)|^2)^{\frac{n}{p}}}.$$

Theorem 2. *Assume that $p > 0$, $u \in H(\mathbb{D}^n)$, μ is a weight on $\mathbb{D}^n$, $\varphi = (\varphi_1, \ldots, \varphi_n)$ is a holomorphic self-map of $\mathbb{D}^n$ and $uC_\varphi : H^p(\mathbb{D}^n) \to H^\infty_\mu(\mathbb{D}^n)$ is bounded. Then*

$$\|uC_\varphi\|_{H^p(\mathbb{D}^n)\to H^\infty_\mu(\mathbb{D}^n)} = \sup_{z\in\mathbb{D}^n} \frac{\mu(z)|u(z)|}{\prod_{j=1}^n (1-|\varphi_j(z)|^2)^{\frac{1}{p}}}.$$

Theorem 3. *Let X be $\mathbb{B}$ or $\mathbb{D}^n$. Assume $p > 0$, $u \in H(X)$ and $M_u : H^p(X) \to H^p(X)$ is bounded. Then*

$$\|M_u\|_{H^p(X)\to H^p(X)} = \|u\|_{H^\infty(X)}.$$

In the proofs of Theorems 1–3 we used the following known lemma ([31,40]):

Lemma 1. *Suppose $p \in (0, \infty)$. Then the following statements hold.*

(a) *For all $f \in H^p(\mathbb{B})$ and $z \in \mathbb{B}$, the following inequality holds*

$$|f(z)| \leqslant \frac{\|f\|_{H^p(\mathbb{B})}}{(1-|z|^2)^{\frac{n}{p}}}.$$

(b) *For all $f \in H^p(\mathbb{D}^n)$ and $z = (z_1, \ldots, z_n) \in \mathbb{D}^n$, the following inequality holds*

$$|f(z)| \leqslant \frac{\|f\|_{H^p(\mathbb{D}^n)}}{\prod_{j=1}^n (1-|z_j|^2)^{\frac{1}{p}}}.$$

Our aim here is to extend Theorems 1–3 for the case of bounded symmetric domains. For this we need a point evaluation estimate in the Hardy space $H^p(\mathcal{D})$.

The following result is a consequence of the main result in [9] where the isometries of $H^p(\mathcal{D})$ space are characterized.

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