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Numerical simulations of traffic data via fluid dynamic approach ☆

S. Blandin [a], G. Bretti [b,*], A. Cutolo [c], B. Piccoli [d]

[a] Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720-1710, USA
[b] Me.Mo.Mat Department of University "La Sapienza", Via A. Scarpa 16, 00161 Rome, Italy
[c] Diima Department of the University of Salerno, Via Ponte Don Melillo, 84084 Fisciano(Sa), Italy
[d] Istituto per le Applicazioni del Calcolo "M. Picone", Viale del Policlinico 137, 00161 Rome, Italy

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ABSTRACT

In this paper we introduce a simulation algorithm based on fluid dynamic models to reproduce the behavior of traffic in a portion of the urban network in Rome. Numerical results, obtained comparing experimental data with numerical solutions, show the effectiveness of our approximation.

1. Introduction

In this paper we develop an algorithm to determine the evolution in time of traffic quantities, such as flux, density and cars' speed. The algorithm is then applied to a portion of the urban area in Rome usually subject to congestion, namely Viale del Muro Torto, for which measured traffic data are available. Two fluid dynamic approaches are considered: the Lighthill–Whitham–Richards (briefly LWR, see [30,32]) model with Newell–Daganzo flux and a phase transition second order model (briefly PT), slight modification of the Colombo model [10]. Our work consists in the calibration of system parameters in such a way to give a good reconstruction of traffic behavior. The numerical scheme used is the Godunov scheme, already proposed for road networks (see [3,4,26,29]). The basic idea is to compute approximate flux and density on a single road, assuming as boundary conditions traffic data measured at the endpoints of it. An estimate of the validity of this procedure is obtained by comparing solutions produced numerically and experimental data detected on the road. Measured data are provided by the municipal society for traffic monitoring and control of Rome, namely ATAC S.p.A. Traffic is observed through an Intelligent Transport System technology, where each subsystem in it, represented by a sensor placed along roads of the city, acquires every minute ($\Delta\tilde{t} = 1$ is the sensor time unit) traffic data such as the flux $\tilde{f}$, the velocity $\tilde{v}$ and the occupation rate $\tilde{o}$. Since each sensor generates a magnetic field, the flux is intended as the number of cars crossing it per minute, the velocity is the average speed of cars at every minute, while the occupation rate is given by the time passed by cars on a sensor, hence it is the time interval in which cars pass through the magnetic field.

Considering the whole road as a sequence of segments, the theory is based on LWR and on phase transition model applied to networks. The LWR theory on networks was proposed and developed in various papers, see [8,9,16,18,21,22,24,28,29]. The network models of transportation systems are assumed to be static in classical approaches, but these models do not allow a correct simulation of heavily congested urban road networks. For this reason, traffic engineers have been studying dynamic traffic assignment or *within-day* models, thus rendering necessary the use of traffic simulation models. Such models, principally created from static network traffic assignments, can be divided in microscopic, mesoscopic and macroscopic (see

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* Corresponding author.
E-mail addresses: blandin@berkeley.edu (S. Blandin), bretti@dmmm.uniroma1.it (G. Bretti), gab.bretti@gmail.com (G. Bretti), a.cutolo@diima.unisa.it (A. Cutolo), b.piccoli@iac.cnr.it (B. Piccoli).

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[1] and the references therein). The main problems of the static approach are that it does not properly reproduce the backward propagation of shocks and the difficulty of collecting experimental data to test the validity of the models.

In the 1950s James Lighthill and Gerald Whitham, two experts in fluid dynamics, and independently Richards, [30–32], thought that the equations describing the flow of water could also describe the flow of car traffic. Fluid dynamic models for traffic flow seem the most appropriate to detect some phenomena as shocks formation and propagation on roads, since solutions can show discontinuities in finite time even starting from smooth initial data (see [2]). This nonlinear formulation, based on the conservation of cars, is given by:

$$\partial_t \rho + \partial_x f(\rho) = 0, \tag{1.1}$$

where $\rho = \rho(t,x)$ is the car density, with $\rho \in [0, \rho_{max}]$, $(t,x) \in \mathbb{R}^2$ and ρ_{max} is the maximum car density. The flux $f(\rho)$ is given by ρv, where v is the average velocity of cars. Assuming v to be a smooth decreasing function of the density ρ, also f depends only on ρ and its graph is called the *fundamental diagram*. We always further assume that f, as function of ρ, is concave.

Many other ideas have been developed by researchers studying traffic from other perspectives, see for instance [12–15,23,25,27,31,33]. In more detail, the procedure followed in our simulations consists of the steps:

(0) data capturing;
(1) data cleaning;
(2) calibration of flux parameters;
(3) generation of approximate solution;
(4) computation of errors.

The main results achieved with the mentioned algorithm are:

- the calibration of traffic parameters, providing the maximum velocity v_{max} very close to measured one $\tilde{v}_{max}$;
- the description of traffic behavior, namely of flux and density, with a good approximation.

Focusing on the latter, the percentage error for LWR model is 10% in the free phase and 17% in the congested phase of traffic, while for PT model one gets 9% in the free phase and 22% in the congested phase. Note that in both cases the percentage error for the congested phase is comparable to detection errors by sensors, which is up to 20%.

Another important byproduct is the reconstruction of the traffic data on the whole road. In particular, this allows to reconstruct the queues evolutions thus permitting a good estimate of the travelling times.

Other numerical algorithms, based on fluid dynamic approaches, were considered by researcher in transportation systems, see for instance [6,13,17,26].

The papers is organized as follows: Sections 2 and 3 are devoted to the description of the mathematical models, while the approximation algorithm is presented in Section 4. In Section 5 the results obtained for the segments composing the road are showed and a comparison between numerical and measured solutions is established. Some animations are reported on the web page [5].

2. LWR on sequence of roads

We consider a long road as a sequence of segments modeled by intervals $[a_i, b_i]$ that meet at some junctions, corresponding to endpoints. In order to describe the evolution in time of traffic we use the LWR model on each segment and describe the dynamics at junctions. A Riemann problem for a scalar conservation law is a Cauchy problem for an initial data of Heaviside type, that is piecewise constant with only one discontinuity. Once Riemann problems are solved, a solution to Cauchy problems can be obtained by wave front tracking, see [2]. Since the flux is concave, the Riemann solutions are of two types: continuous waves called rarefactions and traveling discontinuities called shocks. For a junction, a Riemann problem is a Cauchy problem with an initial data that is constant on each road. Junctions play a fundamental role, as the system at a junction is under-determined, even after prescribing the conservation of cars. Due to finite speed of waves in solutions to (1.1), it is enough assigning the dynamics at each junction separately to obtain an evolution on the whole network.

In our case we have only simple junctions with one incoming and one outgoing road. For instance, consider the case where the incoming road is occupied by cars with maximum density, while the outgoing road is empty, see Fig. 1.

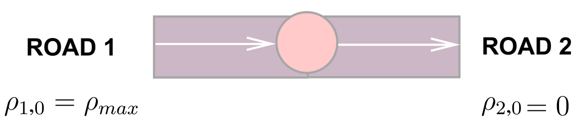

Fig. 1. A particular situation in a junction with one incoming and one outgoing road.