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Peristaltic transport of a Newtonian fluid in a vertical asymmetric channel with heat transfer and porous medium

S. Srinivas *, R. Gayathri

Applied Mathematics Division, VIT University, Vellore 632 014, India

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ABSTRACT

The problem of peristaltic flow of a Newtonian fluid with heat transfer in a vertical asymmetric channel through porous medium is studied under long-wavelength and low-Reynolds number assumptions. The flow is examined in a wave frame of reference moving with the velocity of the wave. The channel asymmetry is produced by choosing the peristaltic wave train on the walls to have different amplitudes and phase. The analytical solution has been obtained in the form of temperature from which an axial velocity, stream function and pressure gradient have been derived. The effects of permeability parameter, Grashof number, heat source/sink parameter, phase difference, varying channel width and wave amplitudes on the pressure gradient, velocity, pressure drop, the phenomenon of trapping and shear stress are discussed numerically and explained graphically.

1. Introduction

Peristaltic pumping is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of distensible duct. In physiology, peristalsis is used by the body to propel or mix the contents of a tube as in ureter, gastro-intestinal tract, bile duct and other glandular ducts. Some worms use peristalsis as a means of locomotion. Roller and finger pumps using viscous fluids also operate on this principle. Peristalsis has been proposed as a mechanism for the transport of spermatozoa in vas deferens. Vas deferens is the duct, which connects the ductus epididymidis to an ampulla. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, blood pumps in heart lung machine and transport of corrosive fluids where the contact of the fluid with the machinery parts is prohibited. Extensive literature on the topic is now available. Some recent interesting investigations in this direction may be given in Refs. [1–20].

The study of fluid flows and heat transfer through porous medium has attracted much attention recently. This is primarily because of numerous applications of flow through porous medium, such as storage of radioactive nuclear waste materials transfer, separation processes in chemical industries, filtration, transpiration cooling, transport processes in aquifers and ground water pollution. Examples of natural porous media are beach sand, sandstone, limestone, rye bread, wood, the human lung, bile duct, gall bladder with stones and in small blood vessels. In some pathological situations, the distribution of fatty cholesterol and artery clogging blood clots in the lumen of coronary artery can be considered as equivalent to a porous medium. Comprehensive literature surveys concerning the subject of porous media can be found in the most recent books by Nield and Bejan [21], Vafai [22], Pop and Ingham [23], Bejan and Kraus [24]. El Shehawey and Husseny [2] formulated a mathematical model for the peristaltic transport of a viscous incompressible fluid through a porous medium bounded by two porous plates. El Shehawey and Sebaei [3] studied the peristaltic mechanism of a Newtonian fluid in a cylindrical tube

* Corresponding author.
E-mail address: srinusuripeddi@hotmail.com (S. Srinivas).

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through a porous medium. Hayat et al. [5] have analyzed the peristaltic flow of a Maxwell fluid including the Hall effect through a porous medium. More recently, Srinivas and Kothandapani [20] investigated the influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls.

Some investigators [25,26] have studied peristalsis with reference to water transport in trees. Though the actual mechanism for the transport of water from the ground to upper branches of tall trees is not well understood, it is speculated that peristalsis and free convection contribute to this motion. The diameters of the trunks of the trees are found to vary with time. Recently, Vajravelu et al. [11] have investigated flow through vertical porous tube with peristalsis and heat transfer. More recently, Mekheimer and Abd elmaboud [14] analyzed the influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus under a zero Reynolds number and long-wavelength approximation. Literature survey witness that the information on heat transfer aspects of peristaltic flow is scant. So far, no investigation is made to the peristaltic flow of viscous fluid in a vertical asymmetric (porous or non-porous) channel with heat transfer. Such consideration is of great value in medical research in understanding heat transfer process in human body. Keeping this in view (and motivated by the previous studies [4,11,14]), a mathematical model is presented to understand the influence of heat transfer on peristaltic transport of a viscous incompressible fluid in a two-dimensional vertical asymmetric porous channel. The momentum and energy equations have been linearized under long-wavelength and low-Reynolds number assumptions and analytical solutions for the flow variables have been derived. The features of the flow characteristics are analyzed by plotting graphs and discussed in detail. The contributions of the porous parameter, in particular, and those of the geometrical parameters, in general, to the flow and heat transfer characteristics are found to be quite significant.

2. Mathematical formulation and solution

We consider the motion of an incompressible viscous fluid in a two-dimensional channel induced by sinusoidal wave trains propagating with constant speed c along the channel walls

$$\begin{aligned} H_1 &= d_1 + a_1 \cos\frac{2\pi}{\lambda}(X - ct) \quad \text{right hand side wall}, \\ H_2 &= -d_2 - b_1 \cos\left(\frac{2\pi}{\lambda}(X - ct) + \varphi\right) \quad \text{left hand side wall}, \end{aligned} \tag{1}$$

where a_1, b_1 are the amplitudes of the waves, λ is the wave length, $d_1 + d_2$ is the width of the channel, the phase difference φ varies in the range $0 \leqslant \varphi \leqslant \pi$, $\varphi = 0$ corresponds to symmetric channel with waves out of phase and $\varphi = \pi$ with waves are in phase, and further a_1, b_1, d_1, d_2 and φ satisfies the condition [4]

$$a_1^2 + b_1^2 + 2a_1 b_1 \cos\varphi \leqslant (d_1 + d_2)^2. \tag{2}$$

The right hand side wall is maintained at temperature T_0 and left hand side wall at T_1 (Fig. 1).

The equation of continuity is

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0. \tag{3}$$

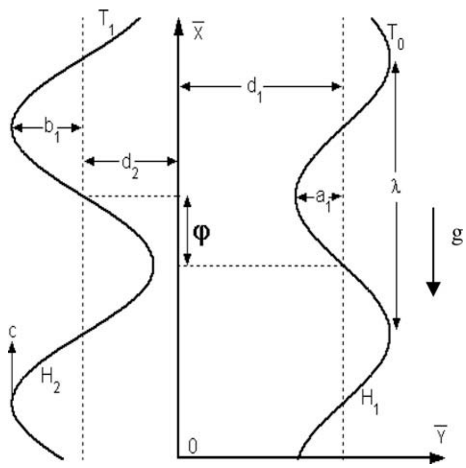

Fig. 1. Flow geometry.