Applied Mathematics and Computation 215 (2009) 308–313

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Optimization models and a GA-based algorithm for stochastic time-cost trade-off problem

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ARTICLE INFO

Keywords:
Time-cost trade-off
Chance-constrained programming
Dependent-chance programming
Stochastic programming
Genetic algorithm

ABSTRACT

In real-life projects, both the trade-off between the project cost and the project completion time, and the uncertainty of the environment are considerable aspects for decision-makers. However, the research on the time-cost trade-off problem seldom concerns stochastic environments. Besides, optimizing the expected value of the objective is the exclusive decision-making criterion in the existing models for the stochastic time-cost trade-off problem. In this paper, two newly developed alternative stochastic time-cost trade-off models are proposed, in which the philosophies of chance-constrained programming and dependent-chance programming are adopted for decision-making. In addition, a hybrid intelligent algorithm integrating stochastic simulations and genetic algorithm is designed to search the quasi-optimal schedules under different decision-making criteria. The goal of the paper is to reveal how to obtain the optimal balance of the project completion time and the project cost in stochastic environments.

1. Introduction

The stochastic time-cost trade-off problem concerns how to modify the project activities in stochastic environments such that the project can be completed in time and meanwhile the project cost can be minimized. It considers the trade-off between the project cost and the project completion time, which is a particular type of project scheduling problem. For project decision-makers, the analysis of the time-cost trade-off is one of the most important aspects of project scheduling and control. In 1961, Kelley [1] first performed research on this special type of project scheduling problem. In the following 40 years, the research on the time-cost trade-off problem mainly focused on the problem with deterministic environments [2,3]. For solving the deterministic time-cost trade-off problem, the common analytical methods are linear programming and dynamic programming [4,5]. Besides, some heuristic algorithms, such as genetic algorithm [6–8], are also introduced.

Although most research on the time-cost trade-off problem assumes that the environment is always deterministic, the real world is full of uncertain factors. The project completion time may be variational due to many external factors, such as the change of weather, the increase of productivity level, etc. Since Freeman [9] introduced the probability theory into the project scheduling problem, many authors have taken into account the nondeterministic factors for characterizing the uncertainty in real projects. Besides, Goldratt [10] questioned the validity of deterministic environments in the project scheduling problem. The reader may also refer to [11–13] to see different types of project scheduling problem with stochastic activity duration times. In 1985, Wollmer [14] discussed a stochastic version of the deterministic linear time-cost trade-off problem, in which some discrete random variables were used to depict the uncertainty in the problem. In 2000, Gutjahr

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doi:10.1016/j.amc.2009.05.004

et al. [15] designed a modified stochastic branch-and-bound approach and applied it to a specific stochastic discrete time-cost trade-off problem. In the existing research of the stochastic time-cost trade-off problem, optimizing the expected objective value is commonly used as the decision-making criterion. Although such a criterion is widely employed in many optimization problems, it does not take into account the probability of disobeying the constraints. For instance, an optimal schedule solved under such a criterion ensures that the expected completion time of the project is under the given time limit, however the probability that the completion time exceeds the time limit can be quite large. In this case, it may be realistic to adopt the optimization philosophy of maximizing the probability of completing the project in time. Therefore, alternative decision-making criteria for the stochastic time-cost trade-off problem are needed, due to the complicated, uncertain environment and the practical demand of diverse optimization goals. Research on investigating alternative decision-making criteria is very useful from both the theoretical and applied points of view.

In this paper, we introduce two new decision-making criteria, chance-constrained programming (CCP) and dependent-chance programming (DCP), to the stochastic time-cost trade-off problem. The activity duration times of a project are described by stochastic variables due to the randomness of the environment. According to the new decision-making criteria, two stochastic models for the time-cost trade-off problem are proposed. A hybrid intelligent algorithm integrating stochastic simulations and genetic algorithm (GA) is designed for solving the stochastic models. Finally, some numerical experiments are performed to reveal the effectiveness of the proposed algorithm.

The remainder of the paper is organized as follows: in Section 2, the stochastic time-cost trade-off problem is described, in which some assumptions and some parameters are given and the formulae of the project completion time and the project cost are deduced. In Section 3, two decision-making criteria are presented, based on which two stochastic time-cost trade-off models are built. In addition, a hybrid intelligent algorithm is designed in this section. The following section shows the effectiveness of the hybrid intelligent algorithm by numerical experiments. Finally, Section 5 draws some conclusions.

2. Problem description

Panagiotakopoulos [16] has shown that the activity duration time and the corresponding activity cost are related with each other. In fact, in most real projects, decision-makers always need to consider the trade-off between the total project cost and the project completion time. Sometimes motivated by reducing the project cost, decisions may be made with the sacrifice of prolonging the project completion time. In other cases, decision-makers may need to amend the schedule of the project to augment the project cost for the demand of finishing the project in time. For example, the decision of hiring more workers can speed up performing the project such that the project can be finished ahead of the due date, however the total project cost increases consequently. Hence it is naturally desirable for decision-makers to find a schedule to complete a project with minimal cost and meanwhile, to satisfy the project completion time constraint.

A project can be represented by an activity-on-the-arc network $G = (V, A)$, where $V = \{1, 2, \ldots, n\}$ is the set of nodes and A is the set of arcs representing the activities. In the network, node 1 and n represent the start and the end of the project, respectively. Fig. 1 shows such a network representing a project. Let the normal duration time of each activity (i,j) be a random variable ξ_{ij} with some given density function, which means that ξ_{ij} represents the duration time of activity (i,j) without the influence of the decision made by the decision-maker. In the project, the uncertainty of ξ_{ij} is derived from the variation of the environment. Correspondingly, the normal cost per day of activity (i,j) is denoted by c_{ij}, which is assumed to be a constant. The decision variable x_{ij} represents the change of the duration time of activity (i,j), which can be controlled by the decision-maker, such as determining the number of workers, determining the quality of instruments, etc. The variable x_{ij}, which is assumed to be an integer for the sake of simplicity, is bounded by some interval $[l_{ij}, u_{ij}]$ owing to some practical conditions, where l_{ij} and u_{ij} are assumed to be integers. Accordingly, for each activity (i,j), there exists another associated cost d_{ij}, which is regarded as the additional cost of per unit change of x_{ij} and is also assumed to be a constant. Then, for the trade-off between the completion time and the project cost, the problem is to decide the optimal vector $\boldsymbol{x} = \{x_{ij} : (i,j) \in A\}$ to meet various scheduling requirements.

The stochastic normal activity duration times are concisely written as $\boldsymbol{\xi} = \{\xi_{ij} : (i,j) \in A\}$. The starting time of activity (i,j) is denoted by $T_{ij}(\boldsymbol{x}, \boldsymbol{\xi})$, and the starting time of activity $(1,j) \in A$ is defined as $T_{1j}(\boldsymbol{x}, \boldsymbol{\xi}) = 0$ since the starting time of the total project is assumed to be 0. For simplicity, we assume that each activity can be processed only if all the foregoing activities are

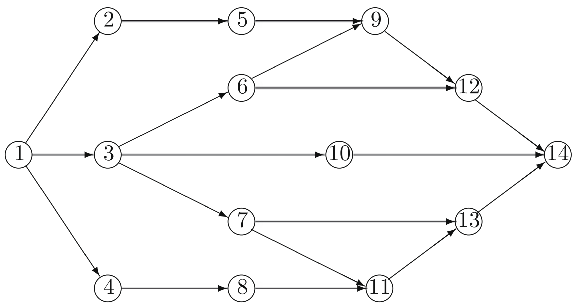

Fig. 1. A project.