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Higher-order symmetric duality in vector optimization problem involving generalized cone-invex functions

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A B S T R A C T

In this paper, a pair of higher-order symmetric dual model in vector optimization problem is formulated. The higher-order cone-pseudoinvex and higher-order strongly cone-pseudoinvex functions are defined. The weak, strong and converse duality theorems are established using these defined functions.

1. Introduction

The vector optimization problem and its dual are said to be symmetric if the dual of the dual is the original problem (see [4]). The notion of symmetric duality was developed significantly by Dantzig et al. [3], Mangasarian [7] formulated a class of second and higher-order dual problems for a non-linear programming problem involving twice differentiable functions.

Higher-order duality in non-linear programming has been studied by several researchers like Chen [2], Mishra [10], Mishra and Rueda [8,9] and Yang et al. [14]. One practical advantage of second and higher-order duality is that it provides tighter bounds for the value of the objective function of the primal problem when approximations are used because there are more parameters involved.

Mishra and Rueda [8] introduced higher-order generalized invexity and duality in mathematical programming. Also, Mishra and Rueda [9] presented higher-order generalized invexity and duality in non-differentiable mathematical programming. Mishra [10] introduced non-differentiable higher-order symmetric duality in mathematical programming with generalized invexity. Zhang [17] presented higher-order convexity and duality in multiobjective programming.

Khurana [6] defined cone-pseudoinvex and strongly cone-pseudoinvex functions and formulated a pair of Mond-Weir type symmetric dual multiobjective programs with established weak duality, strong duality and converse duality theorems under these defined functions. The first-order symmetric and self duality programs in multiobjective non-linear programming problem presented by Kassem [5]. Yang et al. [15] introduced second order generalized convexity and duality in non-differentiable multiobjective mathematical programming. The converse duality in non-linear programming with cone constraints studied by Yang et al. [16].

Recently, Mishra and Giorgi [11] presented an invexity and optimization, non-convex optimization and its applications. Also, Mishra et al. [12] introduced the V-invex functions and vector optimization, optimization and its applications. Suneja et al. [13] introduced a class of higher order (F, ρ, σ)-type I functions for a multiobjective fractional programming problem.

In this paper, a new class of higher-order K-pseudoinvex functions and higher-order strongly K-pseudoinvex functions for a mathematical programming problem is presented. We establish higher-order weak, strong and converse duality theorems under higher-order K-pseudoinvexity assumptions.

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2. Notations and definitions

The higher-order invex functions and higher-order pseudoinvex functions defined by Mishra [10] as a generalization of invex functions are defined as follows:

Definition 1. A differentiable real valued function $f:X \to R$ defined on a non-empty open set $X \subseteq R^n$ is said to be higher-order invex at $u \in X$ with respect to $\eta: X \times X \to R^n$ and $h: X \times R^n \to R$ if $\forall (x,p) \in X \times R^n$,

$$f(x) - f(u) \geqslant \eta^T(x,u)[\nabla_u f(u) + \nabla_p h(u,p)] + h(u,p) - p^T \nabla_p h(u,p).$$

Definition 2. A differentiable real valued function $f:X \to R$ defined on a non-empty open set $X \subseteq R^n$ is said to be higher-order pseudoinvex at $u \in X$ with respect to $\eta: X \times X \to R^n$ and $h: X \times R^n \to R$ if $\forall\ (x,p) \in X \times R^n$,

$$\eta^T(x,u)[\nabla_u f(u) + \nabla_p h(u,p)] \geqslant 0 \Rightarrow f(x) - f(u) - h(u,p) + p^T \nabla_p h(u,p) \geqslant 0.$$

On the lines of Cambini [1], we give the following generalization of higher-order pseudoinvex functions with respect to cones.

Let K be a closed convex pointed cone with $int\ K \neq \phi$.

Definition 3. A differentiable real valued function $f:X \to R$ is said to be higher-order K-pseudoinvex at $u \in X$ with respect to $\eta: X \times X \to R^n$ and $h: X \times R^n \to R$ if $\forall (x,p) \in X \times R^n$,

$$-\eta^T(x,u)[\nabla_u f(u) + \nabla_p h(u,p)] \notin \operatorname{int} K \Rightarrow -[f(x) - f(u) - h(u,p) + p^T \nabla_p h(u,p)] \notin \operatorname{int} K.$$

Definition 4. A differentiable real valued function $f:X \to R$ is said to be higher-order strongly K-pseudoinvex at $u \in X$ with respect to $\eta: X \times X \to R^n$ and $h: X \times R^n \to R$ if $\forall (x,p) \in X \times R^n$,

$$-\eta^T(x,u)[\nabla_u f(u) + \nabla_p h(u,p)] \notin \operatorname{int} K \Rightarrow f(x) - f(u) - h(u,p) + p^T \nabla_p h(u,p) \in K.$$

Remark 1. If $K = R_+$, the higher-order K-pseudoinvex functions and higher-order strongly K-pseudoinvex functions reduce to higher-order pseudoinvex functions defined by Mishra [10].

Remark 2. Every higher-order strongly K-pseudoinvex function is higher-order K-pseudoinvex function but converse is not necessarily true as can be seen from the following example.

Example 1. Let $K = \{(x,y,p): -4x \leqslant y + p\ x, x \geqslant 0, p \geqslant 0\}$ and the functions f, h and η are defined as

$$f(x) = (-x^2 + 2x, e^{-x}), \quad h(x,p) = (x + p^2, -2p) \quad \text{and} \quad \eta(x,u) = x^3 - u.$$

It can be seen that the function f is higher-order K-pseudoinvex at $u = 0$ but f is not higher-order strongly K-pseudoinvex at $u = 0,\ p = \frac{1}{2}$ because for $x = 1$, we have

$$-\eta^T(x,u)[\nabla_u f(u) + \nabla_p h(u,p)] \notin \operatorname{int} K \Rightarrow f(x) - f(u) - h(u,p) + p^T h(u,p) \notin K.$$

We will now give an example of a function, which is higher-order strongly K-pseudoinvex but not higher-order pseudoinvex.

Example 2. Let $K = \{(x,y,p): y + p \geqslant x, y + p \geqslant -x, y \geqslant 0, p \geqslant 0\}$ and the functions

$$f(x) = (-x^2, x^2 + 2x), \quad h(x,p) = (x + p^2, -p) \quad \text{and} \quad \eta(x,u) = x^3 - u.$$

Then the function f is higher-order strongly K-pseudoinvex at $u = 0,\ p = \frac{1}{2}$.

However, the function f is not higher-order pseudoinvex at $u = 0,\ p = \frac{1}{2}$, because for $x = 2$ we have

$$\eta^T(x,u)[\nabla_u f(u) + \nabla_p h(u,p)] \geqslant 0 \Rightarrow f(x) - f(u) - h(u,p) + p^T \nabla_p h(u,p) \ngeqslant 0.$$

Definition 5 [6]. A point $u \in R^n$ is said to be

(i) a minimize of the function f with respect to a closed convex pointed cone K if for every $x \in R^n$, $f(u) - f(x) \notin K \backslash \{0\}$;
(ii) a weak minimize of the function f with respect to a closed convex pointed cone K if for every $x \in R^n$, $f(u) - f(x) \notin \operatorname{int} K$;
(iii) a strong minimize of the function f with respect to a closed convex pointed cone K if for every $x \in R^n$, $f(u) - f(x) \in K$.

Definition 6 [6]. The polar cone K^* of K is defined as

$$K^* = \{z \in R^l : x^T z \geqslant 0 \text{ for all } x \in K\}.$$