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Solutions of the generalized KdV equation with time-dependent damping and dispersion

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ABSTRACT

Solitary wave solutions are obtained for the generalized Korteweg-de-Vries (gKdV) equation with time-dependent damping and dispersion by using the tanh–coth method, the exp-function method and the modified sine–cosine method. These methods are useful and efficient and a variety of solitary wave solutions are obtained that possess variable coefficients.

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1. Introduction

In this paper, we discuss the generalized Korteweg-de-Vries (gKdV) equation with time-dependent damping and dispersion [1] given by

$$u_t + u^n u_x + \alpha(t)u + \beta(t)u_{xxx} = 0, \tag{1}$$

for any positive integer n. Specifically, u_t describes the time evolution and $u^n u_x$ is the nonlinear term. The last two terms represent the effects of linear damping and dispersion with time-dependent coefficients $\alpha(t)$ and $\beta(t)$, respectively. This equation has a wide range of applications and appears, for example, in the study of coastal waves in oceans, in the study of liquid drops and bubbles, and in the investigation of atmospheric blocking phenomena such as dipole blocking [2,3]. The gKdV, however, cannot be integrated by the classical integration methods and so the solitary wave ansatz is used in this paper to demonstrate the effectiveness of the exp-function, the tanh–coth and the sine–cosine methods for the gKdV in the presence of linear time-dependent damping and dispersion.

2. Application of the tanh–coth method

2.1. The tanh–coth method

The standard tanh method [4–6] uses tanh as a new variable because all derivatives of tanh are expressible in terms of tanh. Introducing a new independent variable

$$Y = \tanh(\mu\xi), \quad \xi = x - c(t)t, \tag{2}$$

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transforms the partial differential equation

$$P(u_t, u_x, u_{xx}, u_{xxx}, \ldots\ldots) = 0, \tag{3}$$

into an ordinary differential equation of the wave variable $\xi = x - c(t)t$

$$Q(u, u', u'', u''', \ldots\ldots) = 0. \tag{4}$$

Eq. (4) is then integrated with all the integration constants of the derivative terms taken to be zero. We now apply the change of derivatives

$$\begin{aligned}\frac{d}{d\xi} &= \mu(1 - Y^2)\frac{d}{dY},\\ \frac{d^2}{d\xi^2} &= -2\mu^2 Y(1 - Y^2)\frac{d}{dY} + \mu^2(1 - Y^2)^2\frac{d^2}{dY^2}.\end{aligned} \tag{5}$$

The tanh–coth method [4–6] admits the use of the finite expansion

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^k + \sum_{k=1}^{M} b_k Y^{-k} \tag{6}$$

for some positive integer M, which is known in most cases. The constants a_k and b_k, however, are unknown and have to be determined. Substituting (6) into the ordinary differential equation (4) yields an algebraic equation in the powers of Y and the value of M can then be determined by collecting all the coefficients of the powers of Y in the resulting equation. The vanishing of all these coefficients then yields a system of algebraic equations involving the parameters a_k $(k = 0, \ldots, M)$, μ and $c(t)$.

2.2. Using the tanh–coth method

Use the relation

$$u(\xi) = [\lambda(t)v(\xi)]^{\frac{1}{n}}, \tag{7}$$

to transform (1) to

$$\begin{aligned}&\frac{d\lambda(t)}{dt}v^3 n^2 - [c(t) + \frac{dc(t)}{dt}t]\lambda(t)n^2 v' v^2 + \lambda^2(t)n^2 v' v^3 + \alpha(t)\lambda(t)v^3 n^3 + \lambda(t)\beta(t)[(1-2n)(1-n)(v')^3\\ &\quad + 3n(1-n)vv'v'' + n^2 v''' v^2] = 0.\end{aligned} \tag{8}$$

Then, from $v'v^3$ and $(v')^3$, we have

$$M + 1 + 3M = 3(M + 1), \tag{9}$$

which implies that $M = 2$. So

$$v(x,t) = S(Y) = a_0 + a_1 Y + a_2 Y^2 + b_1 Y^{-1} + b_2 Y^{-2}. \tag{10}$$

Now substitute (10) into (8), collect the coefficients of the powers of Y^i, $-9 \leqslant i \leqslant 9$, set all the coefficients to zero, and solve the resulting system of algebraic equations to obtain the following three possible cases.

Case 1.

$$\begin{aligned}&a_2 = -\frac{2\beta(t)\mu^2(n^2+3n+2)}{n^2\lambda(t)}, \quad \lambda(t) = C_0^n e^{\int -n\alpha(t)dt}, \quad a_0 = \frac{2\beta(t)\mu^2(n^2+3n+2)}{n^2\lambda(t)}, \quad a_1 = 0, b_2 = 0,\\ &b_1 = 0, \quad c(t) = \frac{\int \frac{4\beta(t)\mu^2}{n^2}dt + C_1}{t}.\end{aligned} \tag{11}$$

Case 2.

$$\begin{aligned}&b_2 = -\frac{2\beta(t)\mu^2(n^2+3n+2)}{n^2\lambda(t)}, \quad a_0 = \frac{2\beta(t)\mu^2(n^2+3n+2)}{n^2\lambda(t)}, \quad \lambda(t) = C_0^n e^{\int -n\alpha(t)dt}, \quad a_1 = 0, \quad a_2 = 0,\\ &b_1 = 0, \quad c(t) = \frac{\int \frac{4\beta(t)\mu^2}{n^2}dt + C_1}{t}.\end{aligned} \tag{12}$$

Case 3.

$$\begin{aligned}&a_1 = 0, \quad a_0 = \frac{4\beta(t)\mu^2(n^2+3n+2)}{n^2\lambda(t)}, \quad \lambda(t) = C_0^n e^{\int -n\alpha(t)dt}, \quad c(t) = \frac{\int \frac{16\beta(t)\mu^2}{n^2}dt + C_1}{t},\\ &a_2 = -\frac{2\beta(t)\mu^2(n^2+3n+2)}{n^2\lambda(t)}, \quad b_2 = -\frac{2\beta(t)\mu^2(n^2+3n+2)}{n^2\lambda(t)}, \quad b_1 = 0.\end{aligned} \tag{13}$$

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