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A new one-step smoothing Newton method for nonlinear complementarity problem with P_0-function

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ABSTRACT

In this paper, nonlinear complementarity problem with P_0-function is studied. Based on a new smoothing function, the problem is approximated by a family of parameterized smooth equations and we present a new one-step smoothing Newton method to solve it. At each iteration, the proposed method only need to solve one system of linear equations and perform one Armijo-type line search. The algorithm is proved to be convergent globally and superlinearly without requiring strict complementarity at the solution. Numerical experiments demonstrate the feasibility and efficiency of the new algorithm.

1. Introduction

Consider the the following nonlinear complementarity problem (NCP) of finding a vector $x \in \mathscr{R}^n$ such that

$$x \geqslant 0, \quad f(x) \geqslant 0, \quad x^T f(x) = 0, \tag{1}$$

where $f : \mathscr{R}^n \to \mathscr{R}^n$ is a continuously differentiable P_0-function. For simplicity, the problem defined by (1) is named as P_0-NCP(f).

Recently, there has been strong interests in smoothing method for solving the NCPs [1–9,17–22], and different methods for solving NCPs have been proposed. One motivation is that NCPs have wide applications in many fields [10,11]. The idea of smoothing Newton method is to use a smooth function to reformulate the problem concerned as a family of parameterized smooth equations and solve the smooth equations approximately by using Newton method at each iteration. By reducing the parameter to zero, it is hoped that a solution of the original problem can be found. However, many of these algorithms depend on the assumption of strict complementarity or monotonicity at the KKT points of the problem. Zhang, Han and Huang [7] have proposed a one-step smoothing Newton method for solving the P_0-NCP(f) based on the smoothing symmetric perturbed Fischer–Burmeister (FB) function. Compared to previous literatures, the algorithm has stronger convergence results under weaker conditions. Their algorithm solves only one system of linear equations and performs only one line search per iteration. Lately, Ma and Chen [17] proposed another one-step smoothing Newton method based on a variant smoothing function for NCPs.

Motivated by this direction, in this paper, a smoothing function for P_0-NCP(f) is investigated by smoothing the symmetric perturbed minimum function. Based on this smoothing function, we reformulate the P_0-NCP(f) into a system of nonlinear equations and propose a one-step smoothing Newton method for solving it. It is shown that our algorithm has the following beneficial properties.

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(i) It is shown that this algorithm is well-defined and a solution of P_0-NCP(f) can be obtained from any accumulation point of the iteration sequence generated by the method.
(ii) The algorithm can start from an arbitrary point.
(iii) It need to solve only one system of linear equations and perform only one Armijo-type line search at each iteration.
(iv) The boundedness of the level set can be obtained due to the coerciveness of the smoothing function.
(v) The global and superlinear convergence of the algorithm are obtained without strict complementarity.

This paper is organized as follows. In the next section, we introduce some preliminaries to be used in the subsequent sections and give a new smoothing function and its properties. In Section 3, we present a one-step smoothing Newton method for solving the P_0-NCP(f) and state some preliminary results. The global convergence and superlinear convergence of the algorithm are investigated in Section 4. Numerical results are reported in Section 5. Some conclusions are given in Section 6.

The following notations will be used throughout this paper. All vectors are column vectors, the subscript ^{T} denotes transpose, $\mathscr{R}^n$ (respectively, $\mathscr{R}$) denotes the space of n-dimensional real column vectors (respectively, real numbers), $\mathscr{R}^n_+$ and $\mathscr{R}^n_{++}$ denote the nonnegative and positive orthants of $\mathscr{R}^n$, $\mathscr{R}_+$ (respectively, $\mathscr{R}_{++}$) denotes the nonnegative (respectively, positive) orthant in $\mathscr{R}$. We define $N := \{1,2,\ldots,n\}$. For any vector $u \in \mathscr{R}^n$, we denote by $\mathrm{diag}\{u_i : i \in N\}$ the diagonal matrix whose ith diagonal element is u_i and $\mathrm{vec}\{u_i : i \in N\}$ the vector u. The matrix I represents the identity matrix with suitable dimension. The symbol $\|\cdot\|$ stands for the 2-norm. For any differentiable function $f : \mathscr{R}^n \to \mathscr{R}^n, f'(x)$ denotes the Jacobian of f. For any $\alpha, \beta \in \mathscr{R}_{++}, \alpha = O(\beta)$ (respectively, $\alpha = o(\beta)$) means α/β is uniformly bounded (respectively, tends to zero) as $\beta \to 0$. $\mathscr{R}^n \times \mathscr{R}^m$ is identified with $\mathscr{R}^{n+m}$. For any matrix $A \in \mathscr{R}^{n\times n}, A \succcurlyeq 0$ ($A \succ 0$) means A is positive semi-definite (positive definite, respectively).

2. Preliminaries and a new smoothing function

2.1. Preliminaries

In this subsection, we recall some background materials and preliminary results that will be used in the subsequent sections.

Definition 2.1. A matrix $P \in \mathscr{R}^{n\times n}$ is said to be a P_0-matrix if all its principal minors are nonnegative.

Definition 2.2. A function $f : \mathscr{R}^n \to \mathscr{R}^n$ is said to be a P_0-function if for all $x, y \in \mathscr{R}^n$ with $x \neq y$, there exists an index $i_0 \in N$ such that

$$x_{i0} \neq y_{i0}, \quad (x_{i0} - y_{i0})[f_{i0}(x) - f_{i0}(y)] \geqslant 0.$$

It is noted that $f'(x)$ is a P_0-function if f is a differential P_0 function.

Definition 2.3. Let $\mathscr{Q} \subset \mathscr{R}^n$ be a cone, and $f : \mathscr{Q} \to \mathscr{R}^n$ is a continuous mapping. If there exists a point $u \in \mathscr{Q}$ such that

$$\lim_{\|x\|\to+\infty} \frac{(x-u)^T f(x)}{\|x\|} = +\infty, \quad x \in \mathscr{Q},$$

then the mapping f is called satisfying the coerciveness condition in $\mathscr{Q}$.

Definition 2.4. Suppose that $f : \mathscr{R}^n \to \mathscr{R}^n$ is locally Lipschitz continuous around $x \in \mathscr{R}^n$. f is said to be semi-smooth at x if f is directionally differentiable at x and

$$\lim_{V\in\partial f(x+th'),h'\to h,t\to 0^+} Vh' \quad \text{exists for all } h \in \mathscr{R}^n,$$

where $\partial f(\cdot)$ denotes the generalized derivative in the sense in [12].

The concept of semi-smoothness was originally introduced by Mifflin for functions [16]. Qi and Sun extended the definition of semi-smooth function to vector-valued functions [15]. Convex functions, smooth functions, piecewise linear functions, convex and concave functions, and sub-smooth functions are examples of semi-smooth functions. A function is semi-smooth at x if and only if all its component functions are. The composition of semi-smooth functions is still a semi-smooth function.

2.2. A new smoothing function and its properties

In this subsection, we give a new smoothing function and its properties. For any $(a,b) \in \mathscr{R}^2$, consider the minimum function

$$g(a,b) := \min\{a,b\}. \tag{2}$$