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Residue minimization technique to analyze the efficiency of convective straight fins having temperature-dependent thermal conductivity

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ABSTRACT

In this article, we propose and implement a numerical technique based on residue minimization to solve the nonlinear differential equation, which governs the temperature distribution in straight convective fins having temperature-dependent thermal conductivity. The form of temperature distribution is approximated by a polynomial series, which exactly satisfies the boundary conditions of the problem. The unknown coefficients of the assumed series are optimized using the Nelder–Mead simplex algorithm such that the squared L_2 norm of the residue attains its minimum value within a specified tolerance limit. The near-exact solution thus obtained is further used to calculate the fin efficiency. For the case of constant thermal conductivity, the obtained results are validated with the analytical solutions, while for the case of variable thermal conductivity, the obtained results are corroborated with those previously published in the literature. An excellent agreement in each case consolidates the effectiveness of the proposed numerical technique.

1. Introduction

Extended surfaces (fins) are widely used in many engineering applications which include, but are not limited to, air conditioning, refrigeration, automobile and chemical processing equipments. The primary objective of using fins is to enhance the heat transfer between the base surface and its convective, radiative or convective–radiative environment.

For convective fins having constant thermal conductivity, determination of temperature distribution along the length of the fin requires the solution of a linear differential equation, which is straightforward to obtain [1]. However, when a large temperature gradient exists within the fin, the dependence of thermal conductivity on the temperature can be significant. For most engineering materials, the temperature-dependent thermal conductivity is modeled as a linear function of the temperature [2]. Consequently, the temperature distribution along the length of the fin is governed by a nonlinear differential equation and in most cases, solving these equations involve numerical methods.

The nonlinear fin problem has received a significant attention in recent years in view of its practical applications in semiconductors, heat exchangers, power generators and electronic components. Several researchers have applied well-known numerical techniques to solve the nonlinear fin equation. Some of the pertinent research works include, implementation of the perturbation method [3], Adomian decomposition method [4–6], Taylor's series method [7,8], variational iteration method [9,10], homotopy analysis method [11], homotopy perturbation method [12,13], shooting method [14] and the differential transformation method [15].

Many of the aforementioned numerical techniques comprise an involved procedure to arrive at the final solution. For example, in the Adomian decomposition method (ADM), it is difficult to arrive at the expressions of the Adomian

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polynomials [12]. The variational iteration method is shown to be useful for a lower degree of nonlinearity, however, its performance degrades with an increase in the extent of nonlinearity in the system [10]. In addition, a computer implementation of the aforementioned techniques is relatively difficult, which may be necessary in cases such as, combined heat and mass transfer [8], temperature dependency of heat transfer coefficient [14] and shape optimization of fins [15]. These observations necessitate the development of a simpler yet accurate numerical technique, which is easily programmable and reduces the number of intermediate steps towards obtaining a solution.

To this end, we describe an accurate numerical technique based on residue minimization, which is a two-step process ultimately leading to a near-exact solution to the nonlinear fin equation. The proposed numerical technique relies on approximating the temperature distribution by a polynomial series that exactly satisfies the boundary conditions. The unknown coefficients of the polynomial series are then optimized in order to minimize the squared L_2 norm of the residue. Thus the resulting approximate solution of temperature distribution exactly satisfies all boundary conditions of the problem and near-exactly satisfies the governing differential equation.

The rest of the article is organized in four sections. Section 2 formulates the problem under consideration and highlights the nonlinear nature of the governing differential equation. A two-step solution technique is proposed in Section 3, which results in a near-exact solution to the differential equation. In Section 4, we compare the obtained results with the analytical and the previously published results to bring out the quantitative inferences. The salient conclusions drawn from the present investigation are summarized in Section 5.

2. Formulation of the problem

Fig. 1 shows the schematic of a prismatic fin having an arbitrary cross-sectional area A_c, perimeter p and length b. The temperature of the base surface is T_b, while the ambient temperature is T_a. The tip of the fin $(x = 0)$ is assumed to be insulated. Also, the fin is assumed to be thin, which implies that temperature variations in the longitudinal direction are much larger than those in the transverse direction. The one-dimensional energy balance equation for an infinitesimal small element dx is written as,

$$A_c \frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] - ph(T - T_a) = 0, \tag{1}$$

where h is the convective heat transfer coefficient and $T(x)$ is the temperature along the length of the fin. Eq. (1) is obtained by combining the steady state one-dimensional Fourier heat conduction equation and Newton's law of cooling. While writing this equation, we assume that the heat generation effects are absent, that the radiation from the surface is negligible and that the convective heat transfer coefficient h is uniform over the surface [1]. Also, $k(T)$ denotes the temperature-dependent thermal conductivity, which is approximated by the following relation [2],

$$k(T) = k_a[1 + \bar{\lambda}(T - T_a)], \tag{2}$$

where k_a is the thermal conductivity of the fin at the ambient fluid temperature and $\bar{\lambda}$ is a parameter defining the variation of thermal conductivity. The typical values of the thermal conductivity of metals at normal temperature and pressure (NTP)

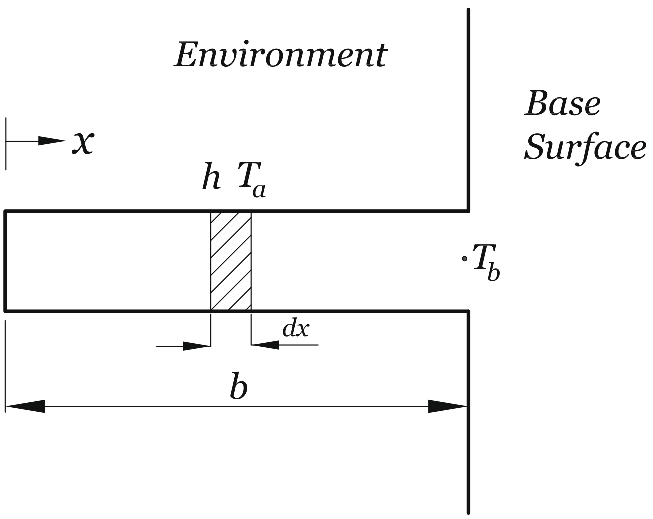

Fig. 1. Schematic of a straight prismatic convective fin.