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Stochastic controllability and minimum energy control of systems with multiple delays in control

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ABSTRACT

In the paper finite-dimensional stationary dynamical control systems described by linear stochastic ordinary differential state equations with multiple point delays in the control are considered. Using notations, theorems and methods taken directly from deterministic controllability problems for linear dynamical systems with delays in control, necessary and sufficient conditions for different kinds of stochastic relative controllability in a given time interval are formulated and proved. It will be proved that under suitable assumptions relative controllability of a deterministic linear associated dynamical system is equivalent to stochastic relative exact controllability and stochastic relative approximate controllability of the original linear stochastic dynamical system. As a special case relative stochastic controllability of dynamical systems with single point delay is also considered. Some remarks and comments on the existing results for stochastic controllability of linear dynamical systems are also presented. In the second part of the paper minimum energy control problem is considered. Under the assumption that system is stochastically relatively exactly controllable analytical formula for minimum energy control is given.

1. Introduction

Controllability is one of the fundamental concept in mathematical control theory and plays an important role both in deterministic and stochastic control theory [6–9,17,19]. Controllability is a qualitative property of dynamical control systems and is of particular importance in control theory. Systematic study of controllability was started at the beginning of 60s, when the theory of controllability based on the description in the form of state space for both time-invariant and time-varying linear control systems was worked out. Roughly speaking, controllability generally means, that it is possible to steer dynamical control system from an arbitrary initial state to an arbitrary final state using the set of admissible controls. In the literature there are many different definitions of controllability, both for linear [6–9,14,19] and nonlinear dynamical systems [11,16,18,20], which strongly depend on class of dynamical control systems and the set of admissible controls [8,10]. Therefore, for deterministic dynamical systems linear and nonlinear there exist many different necessary and sufficient conditions for global and local controllability [6–11].

In recent years various controllability problems for different types of linear dynamical systems have been considered in many publications and monographs. The extensive list of these publications can be found, for example, in the monograph [8] or in the survey papers [9–11]. However, it should be stressed, that the most literature in this direction has been mainly concerned with deterministic controllability problems for finite-dimensional linear dynamical systems with unconstrained controls and without delays.

For stochastic control systems both linear and nonlinear the situation is less satisfactory. In recent years the extensions of the deterministic controllability concepts to stochastic control systems have been recently discussed only in a rather few

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number of publications. In the papers [3–5,14,19,24] different kinds of stochastic controllability were discussed for linear finite-dimensional stationary and nonstationary control systems. The papers [15,17] are devoted to a systematic study of approximate and exact stochastic controllability for linear infinite-dimensional control systems defined in Hilbert spaces. Stochastic controllability for finite-dimensional nonlinear stochastic systems has been discussed in the following papers [1,2,20,22,23]. Using theory of nonlinear bounded operators and linear semigroups different types of stochastic controllability for nonlinear infinite-dimensional control systems defined in Hilbert spaces have been considered in [16,18]. In the papers [12,13] Lyapunov techniques were used to formulate and prove sufficient conditions for stochastic controllability of nonlinear finite-dimensional stochastic systems with point delays in the state variable. Moreover, it should be pointed out, that the functional analysis approach to stochastic controllability problems is also extensively discussed both for linear and nonlinear stochastic control systems in the following papers [15–17,21].

In the present paper we shall study stochastic controllability problems for linear dynamical systems, which are natural generalizations of controllability concepts well known in the theory of infinite-dimensional control systems [8, Chapter 3]; [9]. More precisely, we shall consider stochastic relative exact and approximate controllability problems for finite-dimensional linear stationary dynamical systems with multiple constant point delays in the control described by stochastic ordinary differential state equations. More precisely, using techniques similar to those presented in the papers [14,15,19] we shall formulate and prove necessary and sufficient conditions for stochastic relative exact controllability in a prescribed time interval for linear stationary stochastic dynamical systems with multiple constant point delays in the control.

Roughly speaking, it will be proved that under suitable assumptions relative controllability of a deterministic linear associated dynamical system is equivalent to stochastic relative exact controllability and stochastic relative approximate controllability of the original linear stochastic dynamical system. This is a generalization to control delayed case some previous results concerning stochastic controllability of linear dynamical systems without delays in control [14,15,19].

The paper is organized as follows: Section 2 contains mathematical model of linear, stationary stochastic dynamical system with multiple constant point delays in the control. Moreover, in this section basic notations and definitions of stochastic relative exact and stochastic approximate controllability are presented. Moreover, some preliminary results are also included. In Section 3 using results and methods taken directly from deterministic controllability problems, necessary and sufficient conditions for exact and approximate stochastic relative controllability are formulated and proved. In Section 4, as a special case relative stochastic controllability in a given time interval of dynamical systems with single point delay is also considered. In Sections 5 and 6 minimum energy control problem is formulated and discussed. Under the assumption that system is stochastically relatively exactly controllable analytical formula for minimum energy control is given. Finally, Section 7 contains concluding remarks and states some open controllability problems for more general stochastic dynamical systems.

2. System description

Throughout this paper, unless otherwise specified, we use the following standard notations. Let (Ω, F, P) be a complete probability space with probability measure P on Ω and a filtration $\{F_t | t \in [0, T]\}$ generated by n-dimensional Wiener process $\{w(s)\colon 0 \leqslant s \leqslant t\}$ defined on the probability space (Ω, F, P).

Let $L_2(\Omega, F_T, R^n)$ denotes the Hilbert space of all F_T–measurable square integrable random variables with values in R^n. Moreover, let $L_2^F([0, T], R^n)$ denotes the Hilbert space of all square integrable and F_t-measurable processes with values in R^n.

In the theory of linear, finite-dimensional, time-invariant stochastic dynamical control systems we use mathematical model given by the following stochastic ordinary differential state equation with multiple point delays in the control

$$dx(t) = \left(Ax(t) + \sum_{i=0}^{i=M} B_i u(t - h_i)\right) dt + \sigma\, dw(t) \quad \text{for } t \in [0, T] \tag{1}$$

with initial conditions

$$x(0) = x_0 \in L_2(\Omega, F_T, R^n) \quad \text{and} \quad u(t) = 0 \quad \text{for } t \in [-h_M, 0), \tag{2}$$

where the state $x(t) \in R^n = X$ and the control $u(t) \in R^m = U$, A is $n \times n$-dimensional constant matrix, B_i, $i = 0, 1, 2, \ldots, M$, are $n \times m$-dimensional constant matrices, σ is $n \times n$-dimensional constant matrix, and

$$0 = h_0 < h_1 < \cdots < h_i < \cdots < h_{M-1} < h_M$$

are constant delays.

In the sequel for simplicity of considerations we generally assume that the set of admissible controls $U_{ad} = L_2^F([0, T], R^m)$.

It is well known (see, e.g. [14,15,19] or [20] that for a given initial conditions (2) and any admissible control $u \in U_{ad}$, for $t \in [0, T]$ there exist unique solution $x(t; x_0, u) \in L_2(\Omega, F_t, R^n)$ of the linear stochastic differential state equation (1) which can be represented in the following integral form:

$$x(t; x_0, u) = \exp(At)x_0 + \int_0^t \exp(A(t-s)) \left(\sum_{i=0}^{i=M} B_i u(s - h_i)\right) ds + \int_0^t \exp(A(t-s))\sigma\, dw(s).$$