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Inexact non-interior continuation method for solving large-scale monotone SDCP ☆

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ABSTRACT

For exact Newton method for solving monotone semidefinite complementarity problems (SDCP), one needs to exactly solve a linear system of equations at each iteration. For problems of large size, solving the linear system of equations exactly can be very expensive. In this paper, we propose a new inexact smoothing/continuation algorithm for solution of large-scale monotone SDCP. At each iteration the corresponding linear system of equations is solved only approximately. Under mild assumptions, the algorithm is shown to be both globally and superlinearly convergent.

1. Introduction

In the last serval years, the semidefinite programming problem (SDP) have been studied intensively [3,15–18]. This problem has important applications [9,10], which often call for the solution of large-scale problems. As a natural generalization, the semidefinite complementarity problem (SDCP) is defined as follows [1]: for a given mapping $F : \mathscr{S} \to \mathscr{S} \subseteq \mathscr{X}$, find an $(x, y) \in \mathscr{S} \times \mathscr{S}$ satisfying

$$x \in \mathscr{S}_+, \quad y \in \mathscr{S}_+, \quad \langle x, y \rangle = 0, \quad F(x) = y, \tag{1.1}$$

where $\mathscr{X}$ denote the space of $n \times n$ block-diagonal real matrices with m block of sizes $n_1, \ldots, n_m$, respectively (the block are fixed), $\mathscr{S}$ denote the subspace comprising those $x \in \mathscr{X}$ that are symmetric, that is, $x^T = x$, $\mathscr{S}_+$ denote the convex cone comprising those $x \in \mathscr{S}$ that are positive semidefinite, F is a mapping from $\mathscr{S}$ into itself, and $\langle \cdot, \cdot \rangle$ is the inner product defined by $\langle x, y \rangle := \mathrm{tr}[x^T y]$, where $\mathrm{tr}[\cdot]$ denotes the matrix trace. Similar to [1], in this paper, we assume that F is continuously differentiable monotone function.

A few methods have been developed to solve this problem, such as interior point methods [12], merit function methods [11], and non-interior continuation methods [1,13]. We are interested in non-interior continuation methods for solving monotone SDCP. The pioneer work was done by Chen and Tseng [1]: they studied the existence of Newton directions and boundedness of iterates, and then extended the non-interior continuation method for the NCP to monotone SDCP. The global linear and local superlinear convergence results of the algorithm were proved and the promising numerical results were reported in their paper. After Chen and Tsengs encouraging work, several algorithms and theoretical results for SDCP have been developed [5,13,14].

In exact Newton non-interior continuation method, each iteration consists of finding a solution of linear system of equations which may be expensive if one is solving a large-scale problem. A lot of inexact Newton methods have been proposed

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for solving nonlinear complementarity problem (NCP) [2,7]. In this paper, we will extend inexact Newton methods for solving NCP to large-scale SDCP under the framework of non-interior continuation method. In such an inexact method, the linear system of equations is solved only up to a certain degree of accuracy. The accuracy level of approximate solution is controlled by the so-called forcing parameter which links the norm of residual vector to the norm of mapping at the current iterate. We show that, under mild assumptions, our algorithm is locally superlinearly and even quadratically convergent to a solution of the monotone SDCP.

The paper is organized as follows: in Section 2 we present an inexact non-interior continuation algorithm for solving large-scale monotone SDCP. Convergence results are analyzed in Section 3. Conclusions are given in Section 4.

Throughout this paper, we use the following notation. ":=" means "is defined as". We denote by $\nabla F(x)$ the Jacobian of F at each $x \in \mathscr{S}$, viewed as a linear mapping from $\mathscr{S}$ to $\mathscr{S}$. $\mathscr{S}_{++}$ denote the convex cone comprising those $x \in \mathscr{S}$ that are positive definite. R_+ and R_{++} denote the nonnegative and positive reals. We write $x \succ y$ to mean $x - y$ is positive definite. Landau symbols $o(\cdot)$ and $O(\cdot)$ are defined in usual way. For matrices $x \in \mathscr{S}$, $\|x\|$ is the Frobenius norm. For vector $a \in R^n$, $\|a\|$ denotes 2-norm.

2. Algorithm description

Our inexact non-interior continuation method is based on the following smoothed Fischer–Burmeister function:

$$\phi_\mu(x,y) = x + y - (x^2 + y^2 + 2\mu^2 I)^{\frac{1}{2}}, \tag{2.1}$$

where $(\mu, x, y) \in R \times \mathscr{S} \times \mathscr{S}$ and I is the $n \times n$ identity matrix. This smoothing function was introduced by Kanzow [19] in the case of the NCP based on the Fischer–Burmeister function [20]. Let

$$H_\mu(x,y) := \begin{pmatrix} \phi_\mu(x,y) \\ F(x) - y \end{pmatrix}. \tag{2.2}$$

From Lemma 1 of [1], we know that if $\mu \to 0$, then

$$H_\mu(x,y) \to H_0(x^* y^*) := x^* - [x^* - y^*]_+,$$

where $[x^* - y^*]_+$ denotes the orthogonal projection of $x^* - y^*$ at $\mathscr{S}_+$. Then, by Lemma 2.1 of [11],

$$H_0(x^* y^*) = 0 \Longleftrightarrow (x^* y^*) \quad \text{solves monotone SDCP}.$$

In the remainder of this paper, we will view the number μ as an independent variable. In order to make this clear in our notation, we set $\phi(\mu, x, y) = \phi_\mu(x,y)$, that is

$$\phi(\mu,x,y) = x + y - (x^2 + y^2 + 2\mu^2 I)^{\frac{1}{2}}. \tag{2.3}$$

Further, let

$$H(\mu,x,y) := \begin{pmatrix} \mu \\ F(x) - y \\ \phi(\mu,x,y) \end{pmatrix}. \tag{2.4}$$

In addition, we endow the vector space $R \times \mathscr{S} \times \mathscr{S}$ with the norm

$$|||(\mu,x,y)||| := (|\mu|^2 + \|x\|^2 + \|y\|^2)^{\frac{1}{2}}.$$

Define the merit function $h : R_+ \times \mathscr{S} \times \mathscr{S} \to R_+$ by

$$h(\mu,x,y) = |||H(\mu,x,y)|||^2 = (\mu)^2 + \|F(x) - y\|^2 + \|\phi(\mu,x,y)\|^2.$$

In the remainder of this paper, for the sake of simplicity, denote $z := (\mu, x, y)$, $z^k := (\mu^k, x^k, y^k)$, $\Delta z^k =: (\Delta\mu^k, \Delta x^k, \Delta y^k)$. In each iteration of exact non-interior continuation method for monotone SDCP, linear system of equations

$$\nabla H(z^k)(\Delta z^k) = -H(z^k) \tag{2.5}$$

must be solved exactly, where $\nabla H(z^k)$ denote the Jacobian of H at z^k. However, in each iteration of our method (2.5) is solved only approximately. Our method is to solve the following system of equations

$$\nabla H(z^k)(\Delta z^k) = -H(z^k) + \begin{pmatrix} \hat{\mu}\beta(z^k) \\ \mathbf{r}^k \end{pmatrix}, \tag{2.6}$$

where $\beta(z^k) = \gamma \min\{1, h(z^k)\}$, $\gamma \in (0,1)$ is a constant, $\hat{\mu} \in R_{++}$ is a constant satisfying $\gamma\hat{\mu} < 1$, $\|\mathbf{r}^k\| \leqslant \eta^k |||H(\mu^k, x^k, y^k)|||$, $\eta^k \in (0,1)$. As it is usual in the context of inexact methods, we will refer to the vector $\mathbf{r}^k$ as the residual vector and to the parameter η^k as the forcing term. The forcing term is used to control the level of accuracy in solving the systems equations. We would like to remark that the criterion $\|\mathbf{r}^k\| \leqslant \eta^k |||H(\mu^k, x^k, y^k)|||$ allows us to solve (2.5) to a very low accuracy