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A new method of moving asymptotes for large-scale unconstrained optimization ☆

Haijun Wang [a,b], Qin Ni [a,*]

[a] Nanjing University of Aeronautics and Astronautics, Nanjing 210016, PR China
[b] Science of College, China University of Mining and Technology, Xuzhou 221008, PR China

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ABSTRACT

The method of moving asymptotes is known to work well in the context of structural optimization. A new method of moving asymptotes is proposed for solving large-scale unconstrained optimization problems in this paper. In this method, a descending direction is obtained by solving a convex separable subproblem of moving asymptotes in each iteration. New rules for controlling the asymptotes parameters are designed by using the trust region radius and some approximation properties such that the global convergence of this method is obtained. In addition, a linear search technique is inserted in case of the failure of trust region steps. The numerical results show that the new method may be capable of processing some large-scale problems.

1. Introduction

In this paper, we study the method of moving asymptotes which is designed for solving the following unconstrained optimization problem:

$$\min_{x \in \Re^n} f(x), \tag{1.1}$$

where the function f is assumed to be twicely continuously differentiable in $\Re^n$.

The method of moving asymptotes (MMA, Svanberg, 1987) was first presented in [1]. Afterwards this method was further studied and developed. Ni and Zillober have obtained its globally convergent version by using the trust region and linear searches in [2] and [3], respectively. The MMA has been proven in the past to be an efficient tool to solve structural optimization problems. One of the reasons is the approximation scheme that works very well at least for displacement dependent constraints.

Another reason is that a local optimization model is formulated with only function and gradient evaluations of the current iteration point. For the solution of this local model no further evaluations of the original problem are necessary besides those at the current iteration point. For other extensions and developments of the method of moving asymptotes, see Refs. [1–10].

However, on the one hand, the properties of the moving asymptotes approximation is so far not thoroughly studied, and the choice of the asymptotes is not very reasonable. On the other hand, the MMA has been restricted to at most medium sized problems. The main focus of this paper is to design a new globally convergent MMA method for solving large-scale unconstrained optimization problems. In this method, a descent direction is obtained by solving a convex separable subproblem of moving asymptotes in each iteration. The new rules for controlling the asymptotes parameters are designed by using

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* Corresponding author.
E-mail addresses: hjwcumt@163.com (H. Wang), niqfs@nuaa.edu.cn (Q. Ni).

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the trust region radius and some approximation properties such that the global convergence of the method is obtained. In addition, a linear search technique is inserted in case of the failure of trust region steps.

The paper is organized as follows. In Section 2, a separable subproblem of MMA is discussed in detail. A new algorithm is proposed and its convergence properties are shown in Section 3. Numerical tests are reported in Section 4, and some conclusions are given in the last Section.

2. Separable subproblems

The method of moving asymptotes generally generates a sequence of subproblems. For the problem (1.1), we define its subproblem as follows:

$$\begin{aligned} \min_{x\in\Re^n} \quad & m(x,d) = f(x) + \sum_{i=1}^{n} \phi_i(d_i) \\ \text{s.t.} \quad & |d_i| \leqslant \Delta_i, i = 1,\dots,n \end{aligned} \tag{2.1}$$

where $\Delta_1,\dots,\Delta_n$ are trust region radius,

$$\phi_i(d_i) = \begin{cases} \frac{g_i d_i a_i}{a_i - d_i} + \frac{\varepsilon_i d_i^2}{a_i^2 - d_i^2}, & i \in I_+ \\ \frac{g_i d_i a_i}{a_i + d_i} + \frac{\varepsilon_i d_i^2}{a_i^2 - d_i^2}, & i \in I_- \end{cases}, \tag{2.2}$$

$I_+ = \{i : g_i \geqslant 0,\}, I_- = \{i : g_i < 0,\},\ (g_1,\dots,g_n)^{\mathrm{T}} = \nabla f(x),\ a_i = \Delta_i + \eta_i,\ \eta_i,\ \varepsilon_i$ are positive parameters $\eta_i > 0,\ \varepsilon_i > 0$.

It is easy to see that for $d_i \in [-\Delta_i, \Delta_i],\ i = 1,\dots,n$,

$$a_i - d_i \geqslant \eta_i, \quad a_i^2 - d_i^2 \geqslant \eta_i^2. \tag{2.3}$$

Hence, $\phi_i(d_i)$ is well defined.

It is noted that $\phi_i(d_i)$ in the above formulae (2.2) is different than those in [1–10]. We replace general moving asymptotes with the trust region radius Δ_i and parameter η_i. Thus we can easily give a reasonable way to control the parameters of moving asymptotes.

With some calculations, we have

$$\frac{\partial m(x,d)}{\partial d_i} = \phi_i'(d_i) = \begin{cases} \frac{g_i a_i^2}{(a_i-d_i)^2} + \frac{2\varepsilon_i a_i^2 d_i}{(a_i^2-d_i^2)^2}, & i \in I_+ \\ \frac{g_i a_i^2}{(a_i+d_i)^2} + \frac{2\varepsilon_i a_i^2 d_i}{(a_i^2-d_i^2)^2}, & i \in I_- \end{cases} \tag{2.4}$$

and

$$\frac{\partial^2 m(x,d)}{\partial d_i^2} = \phi_i''(d_i) = \begin{cases} \frac{2g_i a_i^2}{(a_i-d_i)^3} + \frac{2\varepsilon_i a_i^2(a_i^2+3d_i^2)}{(a_i-d_i)^3}, & i \in I_+ \\ \frac{-2g_i a_i^2}{(a_i+d_i)^3} + \frac{2\varepsilon_i a_i^2(a_i^2+3d_i^2)}{(a_i-d_i)^3}, & i \in I_- \end{cases}, \tag{2.5}$$

$$\frac{\partial^2 m(x,d)}{\partial d_i d_j} = 0, \quad i \neq j. \tag{2.6}$$

So we can deduce that the function $m(x,d)$ is a first-order approximation of f, i.e.

$$m(x,0) = f(x), \quad \nabla_d m(x,0) = (\phi_1'(0),\dots,\phi_n'(0))^{\mathrm{T}} = (g_1,\dots,g_n)^{\mathrm{T}} = \nabla f(x).$$

Furthermore, we have $\phi_i''(d_i) > 0$ from (2.4), so it is easy to see that the subproblem (2.1) is separable and convex from (2.2), (2.5) and (2.6). Thus, the subproblem (2.1) is equivalent to n independent one-dimensional bound constrained subproblems

$$\begin{aligned} \min_{d_i} \quad & \phi_i(d_i) \\ \text{s.t.} \quad & d_i| \leqslant \Delta_i, \end{aligned} \tag{2.7}$$

$i = 1,\dots,n$. Because $\phi_i(d_i)$ is a strictly convex function, there exists a unique optimal solution in (2.7).

Let $\phi_i'(d_i) = 0$. From (2.4) we have that for $i \in I_+$

$$\begin{aligned} 0 &= \frac{g_i a_i^2}{(a_i - d_i)^2} + \frac{2\varepsilon_i a_i^2 d_i}{(a_i^2 - d_i^2)^2} \\ &= \frac{a_i^2}{(a_i - d_i)^2}\left[g_i + \frac{\varepsilon_i((a_i + d_i)^2 - (a_i - d_i)^2)}{2a_i(a_i + d_i)^2}\right]. \end{aligned}$$