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Recursive fixed-point smoothing algorithm from covariances based on uncertain observations with correlation in the uncertainty

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ABSTRACT

This paper considers the least-squares linear estimation problem of a discrete-time signal from noisy observations in which the signal can be randomly missing. The uncertainty about the signal being present or missing at the observations is characterized by a set of Bernoulli variables which are correlated when the difference between times is equal to a certain value m. The marginal distribution of each one of these variables, specified by the probability that the signal exists at each observation, as well as their correlation function, are known. A linear recursive filtering and fixed-point smoothing algorithm is obtained using an innovation approach without requiring the state-space model generating the signal, but just the covariance functions of the processes involved in the observation equation.

1. Introduction

There are many practical situations in which the signal appears in the observation in a random manner, for instance, systems where there are intermittent failures in the observation mechanism, fading phenomena in propagation channels, accidental loss of some measurements or inaccessibility of the data at certain times. In this paper the problem of estimating a discrete-time signal from noisy *uncertain observations* in which the signal can be randomly missing is considered. To model the uncertainty, the observation equation, with the usual additive measurement noise, is formulated by multiplying the signal by a binary random variable taking the values one and zero (i.e. a Bernoulli random variable); the value one indicates that the signal is present in the observation, whereas the value zero reflects the fact that the signal is missing. So, the observation equation involves both an additive and a multiplicative noise which models the uncertainty about the signal being present or missing at each observation.

Using a state-space approach, the state estimation problem in discrete-time linear systems with uncertain observations has been widely studied under different hypotheses on additive noises involved in the state and observation equations and, also, under various hypotheses on the multiplicative noise modeling the uncertainty in the observations (see e.g. [5–7] and references therein). However, in some situations, the state-space model of the signal is not available and another type of information must be used to address the estimation problem. In the last years, the signal estimation problem from uncertain observations has been investigated using only the covariance functions of the processes involved in the observation equation

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and algorithms, with a simpler structure than the corresponding ones when the state-space model is known, have been obtained under different hypotheses on the uncertain observation model (see e.g. [3,4] and references therein).

In this paper, we consider the situation of the unknown state-space model; the aim is to construct, using covariance information, linear least-squares estimators of discrete-time signals from uncertain observations verifying that the uncertainty in a time instant k depends only on the uncertainty in the previous time $k-m$. This model for the uncertainty is more general than those considered in [3,4] and this special form of correlation allows us to consider models in which the signal cannot be missing in $m+1$ consecutive observations.

The paper is organized as follows: the next section presents the uncertain observation model and the hypotheses about the processes involved; in Section 3, the filtering and fixed-point smoothing algorithm is derived under an innovation approach; formulae to obtain the estimation error covariance matrices, which provide a global measurement of the estimators performance, are also included in this section. Finally, in Section 4, the algorithm is applied to a simulated observation model where the signal cannot be missing in $m+1$ consecutive observations, situation which can be covered by the general correlation model considered in the theoretical study.

2. Problem statement

In previous papers on signal estimation using covariance information [1,2], the noisy observation is always assumed to contain some information from either the current system output or delayed by one sample time. However, in several cases, observations may contain signal contaminated with noise or noise alone, and only the probability of occurrence of such cases is available for the estimation [3,4]; this is the context in the present paper.

Namely, we consider that each observation y_k is the current n-dimensional random signal, z_k, contaminated by additive noise, v_k, with probability p_k, or noise alone with probability $1-p_k$. To describe this uncertainty, the observation is formulated by multiplying the signal by a Bernoulli random variable θ_k; the value one of this variable indicates that the signal is present in the observation, whereas the value zero reflects the fact that it is missing; so the observation equation is given by

$$y_k = \theta_k z_k + v_k, \quad k \geqslant 1. \tag{1}$$

In some cases, the variables modeling the uncertainty in the observations can be assumed to be independent and, then, their distribution is fully determined by the probability that each observation contains the signal; in this situation, algorithms for the filtering and fixed-point smoothing problems have been derived in [3].

However, there exist many real situations where this independence assumption is not satisfied; for example, in signal transmission models with stand-by sensors in which any failure in the transmission is detected immediately and the old sensor is then replaced, thus avoiding the possibility of the signal being missing in two successive observations. This different situation is analyzed in [4] considering that the Bernoulli variables are correlated at consecutive instants, and the algorithms derived in [3] are extended to this new context.

In this paper, the aim is to obtain recursive algorithms for the least-squares (LS) linear filtering and fixed-point smoothing problems using uncertain observations when the uncertainty in a time instant k depends only on the uncertainty in the previous time $k-m$; this correlation structure, which allows us to consider certain models where the signal cannot be missing in $m+1$ consecutive observations, is more general than that considered in [4] and, consequently, the proposed algorithms generalize those of such paper.

2.1. Model hypotheses

To address the LS linear estimation problem of the signal z_k, it is assumed that the processes involved in (1) satisfy the following hypotheses:

(i) The signal process $\{z_k;\ k \geqslant 1\}$ has zero mean and separable autocovariance function, $K^z_{k,s} = E[z_k z_s^{\mathrm{T}}] = A_k B_s^{\mathrm{T}},\ s \leqslant k$, where A_k and B_s are known $n \times M$ matrices for all $k, s \geqslant 1$.
(ii) The noise process $\{v_k;\ k \geqslant 1\}$ is a zero-mean white sequence with known autocovariance function, $E[v_k v_k^{\mathrm{T}}] = R_k$.
(iii) The multiplicative noise $\{\theta_k;\ k \geqslant 1\}$ is a sequence of Bernoulli random variables with $E[\theta_k] = \bar{\theta}_k$ and autocovariance function $K^\theta_{k,s} = E[(\theta_k - \bar{\theta}_k)(\theta_s - \bar{\theta}_s)] = \delta_{s,k-m} K^\theta_{k,k-m}$, with δ denoting the Kronecker delta function; so $K^\theta_{k,s}$ vanishes for $|k-s| \neq 0, m$, but can be nonzero for $|k-s| = m$.
(iv) The processes $\{z_k;\ k \geqslant 1\}$, $\{v_k;\ k \geqslant 1\}$ and $\{\theta_k;\ k \geqslant 1\}$ are mutually independent.

2.2. General expression of the LS linear estimators

To find the LS linear estimator $\hat{z}_{k/L}$ of z_k given the observations $\{y_1, \ldots, y_L\}$ we use, as in [4], an innovation approach. Specifically, $\{y_1, \ldots, y_L\}$ is transformed to a set of orthogonal vectors, $\{\nu_1, \ldots, \nu_L\}$, named innovations, which is equivalent to the first one in the sense that both sets span the same linear subspace; that is, $\mathscr{L}(\nu_1, \ldots, \nu_L) = \mathscr{L}(y_1, \ldots, y_L)$.

The innovation process is constructed by the *Gram–Schmidt orthogonalization procedure*, using an inductive reasoning. Since the LS linear estimator of a random vector, when any other information is available, is the expectation of such vector,