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On an integral-type operator between bloch-type spaces

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ABSTRACT

Let $H(B)$ denote the space of all holomorphic functions on the unit ball B of $\mathbb{C}^n$. Let φ be a holomorphic self-map of B and $g \in H(B)$, such that $g(0) = 0$. We study the boundedness and compactness of the following integral-type operator recently introduced by Stević

$$P^g_\varphi(f)(z) = \int_0^1 f(\varphi(tz))g(tz)\frac{dt}{t}, \quad z \in B,$$

between Bloch-type spaces. Our main results are natural extensions of some results in the following paper: S. Stević, On a new integral-type operator from the Bloch space to Bloch-type spaces on the unit ball, J. Math. Anal. Appl. 354 (2009) 426–434.

1. Introduction

Let $B^n = B$ be the unit ball of $\mathbb{C}^n$, $B^1 = \mathbb{D}$ the open unit disk in $\mathbb{C}$, $H(B)$ the class of all holomorphic functions on B. For $f \in H(B)$, let

$$\Re f(z) = \sum_{j=1}^n z_j \frac{\partial f}{\partial z_j}(z)$$

represent the radial derivative of f.

For $\alpha > 0$, recall that the α-Bloch space $\mathscr{B}^\alpha = \mathscr{B}^\alpha(B)$, is the space consisting of all functions $f \in H(B)$ such that

$$\|f\|_{\mathscr{B}^\alpha} = |f(0)| + \sup_{z\in B}(1-|z|^2)^\alpha|\Re f(z)| < \infty. \tag{1}$$

Under the above norm, $\mathscr{B}^\alpha$ is a Banach space. When $\alpha = 1$, we get the classical Bloch space $\mathscr{B}$. For more information of the Bloch space and the α-Bloch space (see, e.g., [3,43] and the references therein).

A positive continuous function μ on $[0,1)$ is called normal, if there exist positive numbers s and t, $0 < s < t$, and $\delta \in [0,1)$ such that [26]

$$\frac{\mu(r)}{(1-r)^s} \text{ is decreasing on } [\delta,1) \text{ and } \lim_{r\to1}\frac{\mu(r)}{(1-r)^s} = 0;$$
$$\frac{\mu(r)}{(1-r)^t} \text{ is increasing on } [\delta,1) \text{ and } \lim_{r\to1}\frac{\mu(r)}{(1-r)^t} = \infty.$$

For a normal function ω, an $f \in H(B)$ is said to belong to the Bloch-type space $\mathscr{B}_\omega = \mathscr{B}_\omega(B)$, if

$$\mathscr{B}_\omega(f) := \sup_{z\in B}\omega(|z|)|\Re f(z)| < \infty,$$

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(see, e.g., [39]). The Bloch-type space is a Banach space with the norm

$$\|f\|_{\mathscr{B}_\omega} = |f(0)| + \mathscr{B}_\omega(f).$$

Let $\mathscr{B}_{\omega,0}$ denote the subspace of $\mathscr{B}_\omega$ consisting of those $f \in \mathscr{B}_\omega$ for which

$$\lim_{|z|\to 1} \omega(|z|)|\Re f(z)| = 0.$$

This space is called the little Bloch-type space. When $\mu(r) = (1 - r^2)^\alpha$, the induced spaces $\mathscr{B}_\omega$ and $\mathscr{B}_{\omega,0}$ become the α-Bloch space $\mathscr{B}^\alpha$ and the little α-Bloch space $\mathscr{B}^\alpha_0$.

Let φ be a holomorphic self-map of B. The composition operator C_φ is defined by

$$(C_\varphi f)(z) = (f \circ \varphi)(z), \quad f \in H(B).$$

It is interesting to provide a function theoretic characterization of when φ induces a bounded or compact composition operator on various spaces. Recall that a linear operator is said to be bounded if the image of a bounded set is a bounded set, while a linear operator is compact if it takes bounded sets to sets with compact closure. The book [4] contains plenty of information on this topic. For some recent results in this research area, see, e.g., [3,5,6,20,30,32,40–42,46] and the references therein.

Let $g \in H(\mathbb{D})$ and φ be a holomorphic self-map of $\mathbb{D}$. Products of integral and composition operators on $H(\mathbb{D})$ were introduced by Li and Stević (see, [10,16,17,23], as well as [18,31] for a related operator) as follows:

$$C_\varphi J_g f(z) = \int_0^{\varphi(z)} f(\zeta) g'(\zeta) d\zeta \tag{2}$$

and

$$J_g C_\varphi f(z) = \int_0^z f(\varphi(\zeta)) g'(\zeta) d\zeta. \tag{3}$$

Operators in (2) and (3) are extensions of the following classical integral operator:

$$T_g(f)(z) = \int_0^z f(\zeta) h'(\zeta) d\zeta,$$

which was introduced in [25].

One of the interesting questions has been to extend operators in (2) and (3) in the unit ball settings and to study their function theoretic properties between spaces of holomorphic functions on the unit ball in terms of inducing functions.

If $g \in H(B)$ is such that $g(0) = 0$ and φ is a holomorphic self-map of B, then in [35] Stević introduced the following operator on the unit ball:

$$P^g_\varphi(f)(z) = \int_0^1 f(\varphi(tz)) g(tz) \frac{dt}{t}, \quad f \in H(B), \quad z \in B, \tag{4}$$

and has shown that it is a natural extension of the operator in (3). In [34] Stević proposed a big research project regarding the operator. Some further results in this direction can be found in [36,37]. A particular case of the operator in (4) was independently introduced in [45] and studied in [44].

For $\varphi(z) = z$ and $g \to \Re g$ the operator P^g_φ is reduced to operator T_g, so called, extended Cesàro operator or the Riemann–Stieltjes operator, which was studied, e.g., in [1,2,7–9,11–15,19,21,22,27,39] (see also the references therein). Some related integral-type operators can be found, e.g., in [2,28,29,33,38].

In this paper, we study the boundedness and compactness of the operator P^g_φ between Bloch-type spaces. Our main results are natural extensions of some results in [37]. As a corollary, we obtain some characterizations for the boundedness and compactness of the extended Cesàro operator between Bloch-type spaces.

Throughout this paper C will denote constants in this paper, they are positive and may differ from one occurrence to the other. $a \preceq b$ means that there is a positive constant C such that $a \leqslant Cb$. Moreover, if both $a \preceq b$ and $b \preceq a$ hold, then one says that $a \asymp b$.

2. Auxiliary results

In order to prove our main results, we need some auxiliary results which are incorporated in the following lemmas. The following lemma can be found, for example, in [39,42].

Lemma 1. *Assume that ω is a normal function on $[0, 1)$. If $f \in \mathscr{B}_\omega$, then*

$$|f(z)| \leqslant C\left(1 + \int_0^{|z|} \frac{dt}{\omega(t)}\right) \|f\|_{\mathscr{B}_\omega}$$

for some C independent of f.